Name: Sample Student

Week 2 Sample Talk Comment Form

Date of Talk: 02/29/2020

Title of Talk: "Combinatorial Triangles"

Name of Speaker: Tom Roby

Speaker's Affiliation/Institution: University of Connecticut

Talk Summary: Dr. Roby began by writing some famous sequences on the board, such as the squares, primes, Fibonacci numbers, triangular numbers, and Catalan numbers. After this, he used the projector to show the audience how to use the Online Encyclopedia of Integer Sequences (OEIS) and spoke of its utility in pinpointing a sequence of unfamiliar numbers which might appear in one's own work. (In the sequence A000127, we found... a mistake!)

After this, Dr. Roby wrote down Pascal's triangle on the board. (He also noted in the talk, in multiple places, that other people in history have discovered this triangle, such as Pingala, Khayyam, and YangHui—among them, a poet!) He showed how to construct a row of the triangle by summing adjacent entries in the previous row. Then, he defined the binomial coefficient $\binom{n}{k}$ as the number of ways to choose a k-element subset from an n-element set. He wrote the following relevant identity on the board,

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

and asked if anybody had seen a proof before. A common proof, he said, is by induction—and that it's *terrible*. There other methods of proof, using counting ideas and trickery. Dr. Roby returned to this idea throughout the talk, that induction can often be used to prove something, but is less fun the combinatorial way to do so.

We spent more or less the rest of the talk discussing identities relating to binomial coefficients, such as

$$\sum_{k=0}^{n} \binom{n}{k} = 2^n,$$

and the similar alternating sum

$$\sum_{k=0}^{n} (-1)^k \binom{n}{k} = 0.$$

Each identity may be proved by induction, but Dr. Roby gave interesting binomial proofs for each of them. For the alternating sum, he showed us a method of proof known as "toggling," and also how to prove it using the Binomial Theorem.

We ended the talk by discussing the Entringer numbers, a curious sequence which arises from counting the number of alternating permutations. Dr. Roby showed a way of arranging them in a triangular array, similar to that of Pascal's triangle. The talk was actually supposed to be devoted to numerous examples of this (hence the title), but Dr. Roby decided to explore two examples more closely rather than rush through many examples more shallowly.

Talk Reflection Questions:

1. **What did you find most interesting (mathematically) about the talk?** Please explain your answer.

   My favorite part of the whole talk was when I learned that the Entringer numbers may be used to give a power series expansion for $\sec x$ and $\tan x$, namely via the equality

   $$\sum_{n=1}^{\infty} E_n \frac{x^n}{n!} = \sec x + \tan x,$$

   where E_n denotes the n-th Entringer number. Exploiting the parity of $\sec x$ and $\tan x$ allows us to split this sum over even and odd integers to obtain power series for each function.

   The reason I found this interesting is this: In high school, I tried to find an explicit power series formula for $\tan x$, and it got messy very quickly. I remember looking it up online and couldn't find anything in terms of known numbers (or maybe it was that I asked my teacher and she said no, I don't entirely recall). I wonder if this identity could be use to do evaluate some tricky definite integrals...

2. **Write one or two topics related to this talk that you may be able to write a paper about.** Kind of vague, but I'd be down to write about anything involving binomial coefficients. It would be cool to explore combinatorial proofs of identities involving binomial coefficients. I also liked the method of toggling, and would be interested to learn more about it. The Catalan numbers and Entringer numbers are also things I'd like to know more about.

3. **If you were to write about the mathematical topics written above, what would you need to read and learn more about?** Please explain your answer.

   I think I would need to learn more about the ideas and methods used in combinatorial proofs. I have never studied combinatorics on its own; although, I have seen its shadow in many courses. For a proof method such as toggling, I would need to know more about things that people try to prove with toggling.

4. **Would these make good topics for your final paper? Why?** Give at least two pros and at least two cons to using these topics.

   I think they would be cool topics to write about, on the surface level since I find binomial coefficients and the method of toggling fascinating. It would also be good for my mathematical development, since I have heard combinatorics is one of the easier subjects to self-study. I would also learn about new ways of proving familiar things.

   On the other hand, there is so much to be said about binomial coefficients that I wouldn't want to restrict myself to 10 pages. There is also the issue that such topics seem not-that-related to some of the things I'm learning about right now (e.g. measure theory, rings and modules), and it could be hard to integrate my time if I'm learning about something completely new, especially later in the semester. Of course, all math is kind of related, so I wouldn't discount it completely.