TENTATIVE OUTLINE

AUTHOR

DATE

Abstract

This paper discusses this and that. Leave the abstract blank for now.

Contents

Under each section and subsection, write what you think you may want to research and write about. Cite the references you plan to use in that section/ subsection. Write down what specific facts and concepts you think you may take from each reference.

For example, if you plan to have a section about the Harmonic series and a one-sided bridge, you may want to write what I have written for Section 2. I have included other example sections, Section 3 and Section 4.

1 Introduction

Leave the introduction section blank for now.

2 Tentative Title about the Harmonic Series

2.1 Harmonic Series

In this section, we will discuss the definition of the harmonic series and the fact that the harmonic series is divergent. The source we plan to use is [Ste16, Section 11.2].

2.2 One-sided bridge

We will use the harmonic series to build a one-sided bridge that is as long as we like. We plan to explain the problem of building a one-sided bridge (source: the textbook [Ste16, Problems Plus, Question 12, page 789]) and a solution using the Harmonic Series (source: video [Fow13]).

3 Another Tentative Title about binomial coefficients

In this section, we will study binomial coefficients and q-binomial coefficients.

3.1 binomial coefficients

We will define *binomial coefficients* and study some of its applications. We plan to include a definition, a few examples, one theorem, and one application. We will reference [Bon17, Chapters 3-4] and [Sta11, Chapter 1].

3.2 q-binomial coefficients

We will study the theory of *q-binomial coefficients*, which generalize the binomial coefficients. We plan to include a definition, a few ex-

amples, and two theorems. Our sources are [Sta18, Chapter 6] and [Sta11, Chapter 1].

4 A Third Tentative Title about Catalan numbers

4.1 Catalan numbers

In this section, we will introduce the *Catalan numbers* and a few objects counted by the Catalan numbers. We will give an example (the first ten Catalan numbers), a few formulas for counting the Catalan numbers, and define several objects that are counted by the Catalan numbers: triangulations of a polygon, complete binary trees, and planar binary trees. We will use the books [Sta15], [Sta11], and [Bon17, Section 8.1.2.1]. We will also use the slides [Sta].

4.2 Complete binary trees

This section discusses the *complete binary tree*, one of the many objects which are counted by the Catalan number. We will give a definition of the complete binary trees, write down the number of complete binary trees with $2n+1$ vertices, and display all complete binary trees with $2n+1$ vertices (for small n).

Theorem 4.1 ([Sta15, Chapter 2 question 5])**.** *The number of complete binary trees with* $2n+1$ *vertices (or, equivalently,* $n+1$ *endpoints) is* $\frac{1}{n+1}\binom{2n}{n}$.

4.3 Binary trees and Applications

We will study the applications of binary trees in computing. Our sources will be the wikipedia page [Wik20], the books [Sta15, Sta11], and the slides [Sta].

References

[Bon17] M. Bona. *A walk through combinatorics.* World Scientific Publishing Co., 2017. 4th edition.

[Fow13] Jim Fowler. How far out can you build a one-sided bridge? https://www.youtube.com/watch?v=BjmypdFCl-Q, 2013. [Online; accessed 03-March-2020].

[Sta] Stanley, Richard. Catalan numbers. https://math.mit.edu/~rstan/transparencies/china.pdf. [Online; accessed 03-March-2020].

[Sta11] R. P. Stanley. *Enumerative Combinatorics, Volume 1*. Cambridge University Press, 2011. 2nd edition, http://www-math.mit.edu/~rstan/ec/ec1.pdf.

[Sta15] Richard P. Stanley. *Catalan numbers*. Cambridge University Press, New York, 2015.

[Sta18] R. P. Stanley. *Algebraic combinatorics*. Undergraduate Texts in Mathematics. Springer, 2018. 2nd edition.

[Ste16] James Stewart. *Single variable calculus: Early transcendentals*. Cengage Learning, 2016.

[Wik20] Wikipedia contributors. Binary search tree — Wikipedia, the free encyclopedia. https://en.wikipedia.org/w/index.php?title=Binary_search_tree&oldid=935628330, 2020. [Online; accessed 3-March-2020].