# Outline Assignment 5

The goal of this assignment is to write a tentative outline for your final paper. There will be opportunities to revise your outline later.

a. From egunawan.github.io/writing/resources, **pick between three and five items from "possible paper contents"** that you have already researched (or you think you can finish researching in one month, by the end of March). The items don't need to all come from the same week, but they should fit under one theme. If you choose "others", check with me first before you start researching and writing.

b. Review how to check out electronic books/articles from UConn library and using BibTeX to cite references using MathSciNet and Google Scholar: https://egunawan.github.io/writing/hw/03/guide03.pdf

   If you have trouble getting free access to a reference, please ask me or the math/stat/engineering librarian Renee Walsh: guides.lib.uconn.edu/math.

c. Check out (meaning download, read online, or borrow a paper copy) references as necessary. You must check out at least two, and one of them should be a published book or article.

d. To avoid typos, use Google Scholar or MathSciNet to automatically create a BibTeX item. If you are citing a Wikipedia page, click "cite this page" on the Wikipedia page then select the "BibTeX" option.

e. Create (or ask me to create) a new folder `latex5` in your usual Overleaf project.

f. Use template files

   egunawan.github.io/writing/hw/05/template5.tex (see PDF)
   and
   egunawan.github.io/writing/hw/05/bib5.bib

   to create a new `.tex` file called `outline5.tex` and a new `.bib` file.

g. Look back to the topic items you chose in part (a). Come up with tentative titles for each topic and use these tentative titles as the titles of your sections and subsections. Some papers put a "background" section right after their Introduction.

h. Under each section and subsection, cite the references you plan to use in that section/ subsection. Explain what facts and concepts you think you may take from the references. See the examples I have included in `template5.pdf`

i. **Start writing (Optional)**: Pick one of your sections or subsections, and write a first draft for it. Read the rest of this PDF document for how to structure a content section.

# Writing Content Sections

## 1 Sections and subsections which consist of definitions and examples

- Type up precise definitions that are needed to discuss the concepts, theorems, and questions that you want to write about. Use the definition environment `\begin{definition}`, `\end{definition}`
- After each definition, give examples using `\begin{example}`, `\end{example}`. Start with easy-to-understand examples. For example, if you were to discuss positive Laurent polynomials, you may write the following example and definition.

**Definition 1.1.** A *laurent polynomial* in $x_1, \ldots, x_n$ is a rational function $u$ in $\mathbb{Z}(x_1, \ldots, x_n)$ which can be written as

$$u = \frac{f(x_1, x_2, \ldots, x_n)}{\prod_{i=1}^{n} x_i^{d_i}},$$

where $f$ is a polynomial in $\mathbb{Z}[x_1, x_2, \ldots, x_n]$ and each $d_i$ is a nonnegative integer.

We give examples of Laurent polynomials in two variables.

**Example 1.2.** For example, $u = \frac{b+1}{b}$ and $v = \frac{a+b+1}{ab}$ are two Laurent polynomials in $\mathbb{Z}(a, b)$ with positive coefficients. It is clear that $\frac{v+1}{u}$ is a rational function. Surprisingly,

$$\begin{aligned} \frac{v+1}{u} &= \frac{(\frac{a+b+1}{ab} + 1)}{(\frac{b+1}{b})} \\ &= \frac{a(a+b+1+ab)}{ab(b+1)} \\ &= \frac{a+1}{b}, \end{aligned}$$

so $\frac{v+1}{u}$ is also a Laurent polynomial with positive coefficients.

If you are writing about groups and symmetry, you may include the following example.

**Example 1.3.** The group $C_3$ consists of clockwise rotation by $0^o$, $120^o$, and $240^o$. Figure 1 shows the three configurations of a regular triangle after each rotation.

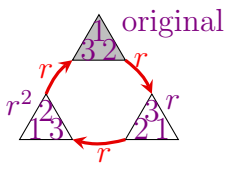


Figure 1: Rotation symmetry of a regular triangle

- If appropriate, you should use subsections to separate multiple topics so that the paper looks organized.

## 2 Sections and subsections containing theorems, conjectures, or open problems

- State proven facts using `\begin{theorem}`, `\end{theorem}` or `\begin{proposition}`, `\end{proposition}`.

> **Proposition 2.1.** *Let $f(x)$ be a single variable polynomial in $\mathbb{R}[x]$ such that $f(x) \geq 0$ for all $x \in \mathbb{R}$. Then $f(x) = g(x)^2 + h(x)^2$ for some polynomials $g(x), h(x) \in \mathbb{R}[x]$.*
>
> **Theorem 2.2** ([Sta15, Chapter 2 question 5])**.** *The number of complete binary trees with $2n+1$ vertices (or, equivalently, $n+1$ endpoints) is $\frac{1}{n+1}\binom{2n}{n}$.*
>
> **Theorem 2.3** (Euclid)**.** *There are infinitely many prime numbers.*

- If you plan on including a short proof or proof sketch, you can give it in this section or in a later section. For example, write:

> We will give a proof of Theorem 2.2 in the next section.
> The following is one of many proofs which have been given for Theorem 2.3.

- If your paper does not include any proof of a theorem you state, then refer the reader to a source that has a proof.
- If appropriate, you should use subsections to separate different but related topics so that the paper looks organized and easy to follow.

## 3 Inserting sentences before and after definition and theorem environment

- At the beginning of each each section or subsection, write at least one *introductory sentence* telling the reader what you will do in this section. For example,

> We state several main theorems related to Hilbert's 17th problems.
>
> This section discusses several of the many objects which are counted by the Catalan number.
>
> In this section, we prove that there are infinitely many prime numbers.
>
> We review basic concepts from group theory. See [Jud19] for a more comprehensive introduction to groups.

- If the entire section/subsection is based on a single source, the first sentence in the section/subsection should make it very clear, for example,

> In this section, we describe several of the 214 Catalan objects discussed in [Sta15].

- If you use two or more sources in this section/subsection, you cite each source either in the theorem or in a sentence immediately before the theorem. Below is an example.

> Every non-negative single variable polynomial can be written as the sum of two squares.
>
> **Proposition 3.1.** *Let $f(x)$ be a single variable polynomial in $\mathbb{R}[x]$ such that $f(x) \geq 0$ for all $x \in \mathbb{R}$. Then $f(x) = g(x)^2 + h(x)^2$ for some polynomials $g(x), h(x) \in \mathbb{R}[x]$.*
>
> The following is a classical proof which was presented during a talk by Antony Rizzie [Riz19].

*Proof.* First, we factor $f(x)$ into irreducible polynomials. ... □

- Write at least one sentence (explaining what is about to happen) immediately before each definition/ example/ definition environment. For example, there is a sentence (written in less-technical language) immediately before Theorem 3.1.

# 4 Further details

This could be one short sentence, a paragraph, or a subsection/section that goes at the end of your paper. For this assignment, your further-details information can one or both of the following:

- One or two open questions related to your paper.
- A sentence pointing the reader to good references if they wish to learn more. To make sure the references are good, you should skim at least the first few pages of the reference.

## 4.1 Examples of an end-of-paper paragraph or sentence

Example 1 (if the paper is about showing that the complete binary trees are counted by the Catalan number, and the paper has *not* cited [Wil05] yet)

> We have provided a generating function proof to show that the complete binary trees are counted by the Catalan number. For more details on the methods of generating function to prove recurrence relations, see [Wil05]. For applications of binary trees in computing, see the wikipedia page [Wik20].

Example 2 (if the paper is about algorithms for finding roots of single variable polynomials, and the paper has not cited [Jud19] yet)

> For more information about polynomial rings, see [Jud19, Chapter 17].

Example 3 (if the paper is about $\pi$ and $e$, and the paper has not stated what is mentioned in the paragraph below yet - it's OK if the paper has cited [Niv05] already)

> We have seen that both $\pi$ and $e$ are irrational. A natural question is to ask whether $\pi/e$ is rational. Surprisingly, it is not yet known whether $\pi$ is a rational multiple of $e$. It is also not known whether $\pi + e$ and $\pi e$ are rational, although it has been proven that at least one of them is rational [Niv05].

- Note: If your writing about open questions is longer than 3 sentences, you should probably create a separate section or subsection for them.

# References


[Jud19] Thomas Judson. *Abstract algebra: theory and applications*. Stephen F. Austin State University, 2019.

[Niv05] Ivan Niven. *Irrational Numbers*. Carus Mathematical Monographs. Mathematical Association of America, 2005.

[Riz19] Anthony Rizzie. Hilbert's 17th problem. https://egunawan.github.io/writing/talks/week6/rizzie.pdf, 2019. Slides from Math Club Talk 2-October-2019, [Online; accessed 03-March-2020].

[Sta15] Richard P. Stanley. *Catalan numbers*. Cambridge University Press, New York, 2015.

[Wik20] Wikipedia contributors. Binary search tree — Wikipedia, the free encyclopedia. https://en.wikipedia.org/w/index.php?title=Binary_search_tree&oldid=935628330, 2020. [Online; accessed 3-March-2020].

[Wil05] Herbert S Wilf. *generatingfunctionology*. AK Peters/CRC Press, 2005.