Assignment 6: Practice Paper

- From egunawan.github.io/writing/resources, **pick between 1 to 4 items from "possible paper contents"** that you have already researched (or you can research in one week). The items don't need to all come from the same week, but they should fit under one theme. If you choose "others", check with me first before you start writing.
- Review how to check out electronic books/articles from UConn library and using BibTeX to cite references using MathSciNet and Google Scholar: egunawan.github.io/writing/hw/05/guide05notes.pdf

  If you have trouble getting free access to a reference, please ask me or the math/stat/engineering librarian Renee Walsh: guides.lib.uconn.edu/math.
- Check out (meaning download, read online, or borrow a paper copy) references as necessary. You must check out at least two, and one of them should be a published book or article.
- To avoid typos, use Google Scholar or MathSciNet to automatically create a BibTeX item. If you are citing a Wikipedia page, click "cite this page" on the Wikipedia page then select the "BibTeX" option.
- To submit the project, simply create a new folder `assignment6` in your usual Overleaf project by the deadline.
- Use template files egunawan.github.io/writing/hw/06/papertemplate6.tex, PDF and egunawan.github.io/writing/hw/06/paper6.bib to create a new `.tex` file called `paper6.tex` and a new `.bib` file.
- Write the first draft of a short paper (2-4 pages). If you write a lot, you can uncomment `\usepackage{fullpage}` which reduces the margin so that your writing fits into fewer pages.
- Note that you may copy and paste this short paper into your final paper. Please read the rest of this document for what should go into assingment 6 paper.

1 Section 1: Introduction

- Using the books/articles you have checked out, find answers to "why do we care?" Usually these can be found in the introduction paragraph or the preface of a book. A possible motivation is connection to natural science or social science; connection to applied mathematics (finance, security, data science, economics, etc); connection to another (possibly theoretical) area of mathematics that many mathematicians are interested in; or simply because people have been fascinated by this topic for a long time.
- If you cannot find any information about why people care about this topic, maybe you should choose a different topic.
- (Optional) Write historical anecdotes and facts: when and where questions were introduced, when people found applications to other areas, when theorems were proved. If the concepts are not from ancient time, discuss the individuals and teams who first came up with the concepts and who made significant contributions to this area
- The current convention of writing math papers is to cite your sources either at the end of your sentence or as part of your sentence (and use the word like "in" or "read" or "see"), but never at the beginning of a sentence. For example, both of these are acceptable.

  There are over 214 known combinatorial interpretations of Catalan numbers. For a list of at least 214 of them, see [Sta15].

  There are over 214 known combinatorial interpretations of Catalan numbers [Sta15].

  Both these paragraphs are acceptable.

  According to [Sta15, Appendix B.1 by Igor Pak], the first known appearance of the Catalan number was in a 1730s book *Quick Methods for Accurate Values* by a Mongolian scientist and mathematician Ming Antu, but the book's connection to Catalan numbers was only observed in 1988 by Luo Jianjin [Luo88]. See [Luo13] for more information.

  The Catalan numbers have appeared in the literature since the 19th century under various names, but the name "Catalan numbers" gained wide acceptance after a popular 1968 publication by a US combinatorialist John Riordan [Sta15, Appendix B.9 by Igor Pak]. In 1988, it was discovered that the Catalan numbers have been studied by a Mongolian mathematician and scientist Ming Antu as early as the 1730s [Luo88, Luo13].

2 Sections and subsections which consist of definitions and examples

- Type up precise definitions that are needed to discuss the concepts, theorems, and questions that you want to write about. Use the definition environment `\begin{definition}`, `\end{definition}`
- After each definition, give examples using `\begin{example}`, `\end{example}`. Start with easy-to-understand examples. For example, if you were to discuss positive Laurent polynomials, you may write the following example and definition.

Definition 2.1. A *laurent polynomial* in $x_1, \ldots, x_n$ is a rational function u in $\mathbb{Z}(x_1, \ldots, x_n)$ which can be written as

$$u = \frac{f(x_1, x_2, \ldots, x_n)}{\prod_{i=1}^{n} x_i^{d_i}},$$

where f is a polynomial in $\mathbb{Z}[x_1, x_2, \ldots, x_n]$ and each d_i is a nonnegative integer.

We give examples of Laurent polynomials in two variables.

Example 2.2. For example, $u = \frac{b+1}{b}$ and $v = \frac{a+b+1}{ab}$ are two Laurent polynomials in $\mathbb{Z}(a, b)$ with positive coefficients. It is clear that $\frac{v+1}{u}$ is a rational function. Surprisingly,

$$\begin{aligned} \frac{v+1}{u} &= \frac{(\frac{a+b+1}{ab} + 1)}{(\frac{b+1}{b})} \\ &= \frac{a(a+b+1+ab)}{ab(b+1)} \\ &= \frac{a+1}{b}, \end{aligned}$$

so $\frac{v+1}{u}$ is also a Laurent polynomial with positive coefficients.

If you are writing about groups and symmetry, you may include the following example.

Example 2.3. The group C_3 consists of clockwise rotation by 0^o, 120^o, and 240^o. Figure 1 shows the three configurations of a regular triangle after each rotation.

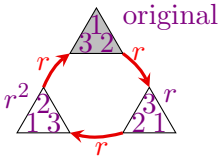

Figure 1: Rotation symmetry of a regular triangle

- If appropriate, you should use subsections to separate multiple topics so that the paper looks organized.

3 Sections and subsections containing theorems, conjectures, or open problems

- State proven facts using `\begin{theorem}`, `\end{theorem}` or `\begin{proposition}`, `\end{proposition}`.

Proposition 3.1. *Let $f(x)$ be a single variable polynomial in $\mathbb{R}[x]$ such that $f(x) \geq 0$ for all $x \in \mathbb{R}$. Then $f(x) = g(x)^2 + h(x)^2$ for some polynomials $g(x), h(x) \in \mathbb{R}[x]$.*

Theorem 3.2 ([Sta15, Chapter 2 question 5])**.** *The number of complete binary trees with $2n+1$ vertices (or, equivalently, $n+1$ endpoints) is $\frac{1}{n+1}\binom{2n}{n}$.*

Theorem 3.3 (Euclid)**.** *There are infinitely many prime numbers.*

- If you plan on including a short proof or proof sketch, you can give it in this section or in a later section. For example, write:

  We will give a proof of Theorem 3.2 in the next section.
  The following is one of many proofs which have been given for Theorem 3.3.

- If your paper does not include any proof of a theorem you state, then refer the reader to a source that has a proof.
- If appropriate, you should use subsections to separate different but related topics so that the paper looks organized and easy to follow.

4 Inserting sentences before and after definition and theorem environment

- At the beginning of each each section or subsection, write at least one *introductory sentence* telling the reader what you will do in this section. For example,

  We state several main theorems related to Hilbert's 17th problems.

  This section discusses several of the many objects which are counted by the Catalan number.

  In this section, we prove that there are infinitely many prime numbers.

  We review basic concepts from group theory. See [Jud19] for a more comprehensive introduction to groups.

- If the entire section/subsection is based on a single source, the first sentence in the section/subsection should make it very clear, for example,

  In this section, we describe several of the 214 Catalan objects discussed in [Sta15].

- If you use two or more sources in this section/subsection, you cite each source either in the theorem or in a sentence immediately before the theorem. Below is an example.

  Every non-negative single variable polynomial can be written as the sum of two squares.

  Proposition 4.1. *Let $f(x)$ be a single variable polynomial in $\mathbb{R}[x]$ such that $f(x) \geq 0$ for all $x \in \mathbb{R}$. Then $f(x) = g(x)^2 + h(x)^2$ for some polynomials $g(x), h(x) \in \mathbb{R}[x]$.*

  The following is a classical proof which was presented during a talk by Antony Rizzie [Riz19].

  Proof. First, we factor $f(x)$ into irreducible polynomials. . . . □

- Write at least one sentence (explaining what is about to happen) immediately before each definition/ example/ definition environment. For example, there is a sentence (written in less-technical language) immediately before Theorem 4.1.

5 Conclusion

Write a conclusion. This could be one sentence, one paragraph, or one subsection/section. For this assignment, your conclusion should contain one or both of the following:

- One or two open questions related to your paper.
- A sentence pointing the reader to good references if they wish to learn more. To make sure the references are good, you should skim at least the first few pages of the reference.

5.1 Examples of a conclusion paragraph or sentence

Example 1 (if the paper is about showing that the complete binary trees are counted by the Catalan number, and the paper has *not* cited [Wil05] yet)

> We have provided a generating function proof to show that the complete binary trees are counted by the Catalan number. For more details on the methods of generating function to prove recurrence relations, see [Wil05]. For applications of binary trees in computing, see the wikipedia page [Wik19].

Example 2 (if the paper is about algorithms for finding roots of single variable polynomials, and the paper has not cited [Jud19] yet)

> For more information about polynomial rings, see [Jud19, Chapter 17].

Example 3 (if the paper is about π and e, and the paper has not stated what is mentioned in the paragraph below yet - it's OK if the paper has cited [Niv05] already)

> We have seen that both π and e are irrational. A natural question is to ask whether π/e is rational. Surprisingly, it is not yet known whether π is a rational multiple of e. It is also not known whether $\pi + e$ and πe are rational, although it has been proven that at least one of them is rational [Niv05].

- Note: If your writing about open questions is longer than 3 sentences, you should probably create a separate section or subsection for them.

6 Paper outline paragraph

- The last paragraph of your introduction is your "paper outline paragraph." It should refer to each section (and subsection, if important enough) using `\ref`. The following are possible ways to start the paragraph.
  - The rest of the paper is organized as follows. In Section 2, we ...
  - This paper is organized as follows. We ... in Section 2.1.
  - We begin by ... in Section 2. In Section 3, we ...
- See the last paragraph of the introduction of the sample student papers posted in egunawan.github.io/writing/resources

7 Abstract

Wait until you finish writing the paper before writing your abstract. Briefly state the main statements and goals of the body of the paper. If you paper is only 2 pages, then your abstract should probably contain one or (at most) two sentences.

Do *not* use the `\cite` or `\ref` in an abstract, but it's OK to spell out the names of the authors of your references if they play a big role in your paper.

References

[Jud19] Thomas Judson. *Abstract algebra: theory and applications*. Stephen F. Austin State University, 2019.

[Luo88] Jianjin Luo. The first inventor of Catalan numbers in the world. *Neimenggu Daxue Xuebao*, 19:239–245, 1988.

[Luo13] Jianjin Luo. Ming Antu and his power series expansions. In *Seki, founder of modern mathematics in Japan*, volume 39 of *Springer Proc. Math. Stat.*, pages 299–310. Springer, Tokyo, 2013.

[Niv05] Ivan Niven. *Irrational Numbers*. Carus Mathematical Monographs. Mathematical Association of America, 2005.

[Riz19] Anthony Rizzie. Hilbert's 17th problem. https://egunawan.github.io/writing/talks/week6/rizzie.pdf, 2019. Slides from Math Club Talk 2-October-2019, [Online; accessed 19-October-2019].

[Sta15] Richard P. Stanley. *Catalan numbers*. Cambridge University Press, New York, 2015.

[Wik19] Wikipedia contributors. Binary search tree — Wikipedia, the free encyclopedia. https://en.wikipedia.org/w/index.php?title=Binary_search_tree&oldid=922009718, 2019. [Online; accessed 19-October-2019].

[Wil05] Herbert S Wilf. *generatingfunctionology*. AK Peters/CRC Press, 2005.