

Frieze vectors and unitary friezes

The identity frieze for the type A_3 quiver $Q = 1 \rightarrow 2 \leftarrow 3$

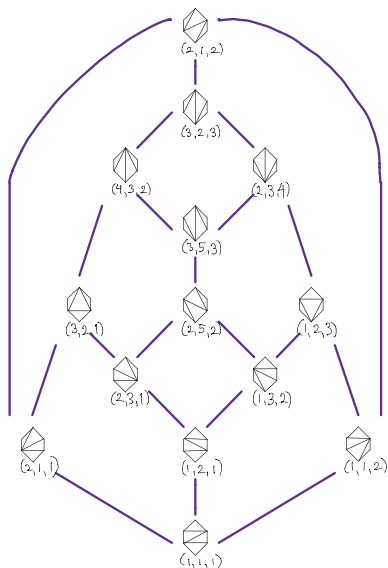
1	1	1	1	
	x_3	$\frac{x_1 x_3 + 1 + x_2}{x_2 x_3}$	$\frac{x_2 + 1}{x_1}$	x_1
x_2	$\frac{x_1 x_3 + 1}{x_2}$	$\frac{x_2^2 + 2x_2 + 1 + x_1 x_3}{x_1 x_2 x_3}$	x_2	x_2
	x_1	$\frac{x_1 x_3 + 1 + x_2}{x_1 x_2}$	$\frac{x_2 + 1}{x_3}$	x_3
1	1	1	1	

Positive integral friezes

Setting $x_1 = x_2 = x_3 = 1$ produces a Conway – Coxeter frieze pattern

1	1	1	1	
	1	3	2	1
1	2	5	1	
	1	3	2	1
1	1	1	1	

- ▶ The above frieze corresponds to the frieze vector $(1, 1, 1)$ relative to $Q = 1 \rightarrow 2 \leftarrow 3$.
- ▶ Given any type A_3 quiver, there are 14 integer frieze vectors (whose values depend on the quiver).



Frieze vectors relative to $Q = 1 \rightarrow 2 \leftarrow 3$

Frieze vectors and unitary friezes

Up to symmetry, there are exactly 2 positive friezes of type $\tilde{\mathbb{A}}_{1,2}$.

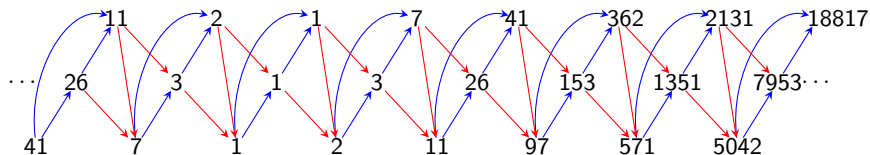


Figure: An $\tilde{\mathbb{A}}_{1,2}$ frieze obtained by specializing the cluster variables of an acyclic seed to 1. The two peripheral arcs have frieze values 2 and 3.

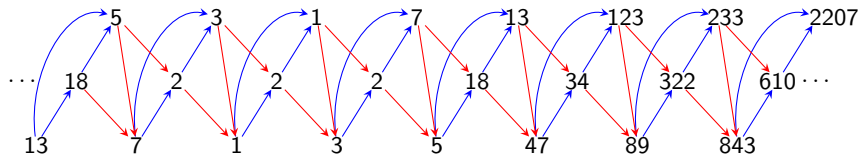


Figure: An $\tilde{\mathbb{A}}_{1,2}$ frieze obtained by specializing the cluster variables of a non-acyclic seed to 1. The two peripheral arcs have frieze values 1 and 5.