First & Last Name: ____________________ Student ID: ____________

Linear Algebra Worksheet Section 4.2

1 T/F Questions

Let $A = \begin{bmatrix} 1 & -2 & -1 & 3 \\ 2 & -4 & 1 & 0 \\ 1 & -2 & 2 & -3 \end{bmatrix}$.

(a) True or false? The set of all solutions to $A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ is a subspace of $\mathbb{R}^4$. ◯ True ◯ False

Briefly (in 1-2 sentences) justify why:

(b) True or false? The set of all solutions to $A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 2 \end{bmatrix}$ is a subspace of $\mathbb{R}^4$. ◯ True ◯ False

Briefly justify why:

(c) Let H be the set of all vectors in $\mathbb{R}^4$ whose entries a, b, c, d satisfy the equations $a - 2b + 5c = d$ and $c - a = b$.

True or false? H is a subspace of $\mathbb{R}^4$. ◯ True ◯ False

Briefly justify why:

(d) Let H be the set of all vectors in $\mathbb{R}^4$ whose entries a, b, c, d satisfy the equations $a - 2b + 5c = d + 1$ and $c - a = b$.

True or false? H is a subspace of $\mathbb{R}^4$. ◯ True ◯ False

Briefly justify why:

Recall that $\mathbb{P}_2$ denotes the vector space of polynomials whose degree is at most 2.

Def. The kernel of a linear map T from V to W is the set of elements in the domain V that are sent by T to the zero object in the codomain W.

2 The kernel and range (image) of a linear transformation

Define a linear map $T : \mathbb{P}_2 \to \mathbb{R}^3$ by

$$T(\mathbf{p}) = \begin{bmatrix} \mathbf{p}(5) \\ \mathbf{p}(5) \\ \mathbf{p}(5) \end{bmatrix}$$

For example, if $\mathbf{p}(t) = 3 + t^2$, then $T(\mathbf{p}) = \begin{bmatrix} 28 \\ 28 \\ 28 \end{bmatrix}$.

(a) What is the kernel of T? Find two polynomials in $\mathbb{P}_2$ that span the kernel of T.

(b) What is the range of T? Find a vector in $\mathbb{R}^3$ which spans the range of T.