First & Last Name: _____ Student ID: _____

Linear Algebra Worksheet Sec 4.1

1 Closed vs not closed under scalar multiplication

1.1 Definition

What does it mean for a set K to be closed under scalar multiplication?

Solution: For every element x in K and every scalar c in $\mathbb{R}$, the scalar multiple cx is in K.

1.2 Not closed under scalar multiplication

Let H be the set of all points in $\mathbb{R}^2$ of the form $\begin{bmatrix} 3s \\ 2+5s \end{bmatrix}$ for $s \in \mathbb{R}$. Show that H is not closed under scalar multiplication. (Find a specific vector $\mathbf{u}$ in H and a specific scalar c such that $c\mathbf{u}$ is not in H.)

Solution: See the "Solution to Practice Problems" at the end of Section 4.1 in the textbook (take $u = \left[\begin{smallmatrix} 3 \\ 7 \end{smallmatrix}\right]$ and take $c = 2$.

Alternatively, take $s = 0$ and $c = 2$.

Note: This also shows that H is *not* a subspace (since a subspace is closed under scalar multiplication).

Solution: This page will show up again when we get to Section 4.2.

Def. The kernel of a linear map T from V to W is the set of elements in the domain V that are sent by T to the zero object in the codomain W.

2 The kernel and range (image) of a linear transformation

Define a linear map $T : \mathbb{P}_2 \to \mathbb{R}^3$ by

$$T(\mathbf{p}) = \begin{bmatrix} \mathbf{p}(5) \\ \mathbf{p}(5) \\ \mathbf{p}(5) \end{bmatrix}$$

For example, if $\mathbf{p}(t) = 3 + t^2$, then $T(\mathbf{p}) = \begin{bmatrix} 28 \\ 28 \\ 28 \end{bmatrix}$.

(a) What is the zero object in $\mathbb{R}^3$? What is the kernel of T? Find two polynomials in $\mathbb{P}_2$ that span the kernel of T.

(b) What is the range of T? Find a vector in $\mathbb{R}^3$ which spans the range of T.