

Linear Algebra Worksheet Sec 4.1**1 Closed vs not closed under scalar multiplication****1.1 Definition**

What does it mean for a set K to be closed under scalar multiplication?

1.2 Not closed under scalar multiplication

Let H be the set of all points in $\mathbb{R}^2$ of the form $\begin{bmatrix} 3s \\ 2 + 5s \end{bmatrix}$ for $s \in \mathbb{R}$. Show that H is not closed under scalar multiplication. (Find a specific vector $\mathbf{u}$ in H and a specific scalar c such that $c\mathbf{u}$ is not in H .)

Note: This also shows that H is *not* a subspace (since a subspace is closed under scalar multiplication).

Def. The kernel of a linear map T from V to W is the set of elements in the domain V that are sent by T to the zero object in the codomain W .

2 The kernel and range (image) of a linear transformation

Define a linear map $T : \mathbb{P}_2 \rightarrow \mathbb{R}^3$ by

$$T(\mathbf{p}) = \begin{bmatrix} \mathbf{p}(5) \\ \mathbf{p}(5) \\ \mathbf{p}(5) \end{bmatrix}$$

For example, if $\mathbf{p}(t) = 3 + t^2$, then $T(\mathbf{p}) = \begin{bmatrix} 28 \\ 28 \\ 28 \end{bmatrix}$.

(a) What is the zero object in $\mathbb{R}^3$? What is the kernel of T ? Find two polynomials in $\mathbb{P}_2$ that span the kernel of T .

(b) What is the range of T ? Find a vector in $\mathbb{R}^3$ which spans the range of T .