Math 2220 Linear Algebra Ch 1 Review

1 Section 1.1

Consider the system (of linear equations)

$$\begin{array}{rrrrl} x_1 & -2x_2 & -x_3 & +3x_4 & = 1 \\ 2x_1 & -4x_2 & +x_3 & & = 5 \\ x_1 & -2x_2 & +2x_3 & -3x_4 & = 4 \end{array} \tag{1.1}$$

2 Sec 1.3 Vector equations

Write an vector equation that is equivalent to the system (1.1)

3 Sec 1.4 The matrix equation $A\mathbf{x} = \mathbf{b}$

Write a matrix equation that is equivalent to the system (1.1)

Let $A = \begin{bmatrix} 1 & -2 & -1 & 3 \\ 2 & -4 & 1 & 0 \\ 1 & -2 & 2 & -3 \end{bmatrix}$.

4 Sec 1.1 Systems of linear equations and Sec 1.2 Row reduction and echelon forms

Is the system from (1.1), $\begin{cases} x_1 & -2x_2 & -x_3 & +3x_4 & = 1 \\ 2x_1 & -4x_2 & +x_3 & & = 5 \\ x_1 & -2x_2 & +2x_3 & -3x_4 & = 4 \end{cases}$ consistent or inconsistent?

Fact: A system of linear equations has ...

1. no solutions, or
2.
3.

(Con't answering Question 4)

The augmented matrix of the system (1.1) is

$$\left[\begin{array}{cccc|c} 1 & -2 & -1 & 3 & 1 \\ 2 & -4 & 1 & 0 & 5 \\ 1 & -2 & 2 & -3 & 4 \end{array}\right] \tag{4.1}$$

Row reduce:

An echelon form of the augmented matrix (4.1) gives us enough information to determine whether a system is consistent/inconsistent because of the following theorem.

Theorem:
A linear system is ________________ if and only if

Since an echelon form of the augmented matrix (4.1) __________________________, the system (1.1) is ________________.

Optional: Continue to row reduce to obtain the UNIQUE __________________________________:

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 1 & 2 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

5 Sec 1.3 and 1.4

Is $\begin{bmatrix}1\\5\\4\end{bmatrix}$ in the subset of $\mathbb{R}^3$ spanned by the columns of $A = \begin{bmatrix}1 & -2 & -1 & 3\\ 2 & -4 & 1 & 0\\ 1 & -2 & 2 & -3\end{bmatrix}$? Why/why not?

Let $\mathbf{v}_1 = \left[\begin{smallmatrix}1\\2\\1\end{smallmatrix}\right]$, $\mathbf{v}_2 = \left[\begin{smallmatrix}-2\\-4\\-2\end{smallmatrix}\right]$, $\mathbf{v}_3 = \left[\begin{smallmatrix}-1\\1\\2\end{smallmatrix}\right]$, and $\mathbf{v}_4 = \left[\begin{smallmatrix}3\\0\\-3\end{smallmatrix}\right]$.
The subset of $\mathbb{R}^3$ spanned (or generated) by $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$, denoted

______________________________,

is ...

6 Sec 1.5 Solution sets of linear systems

(a) Describe all solutions of the ______________________ system

$$\begin{bmatrix} 1 & -2 & -1 & 3 \\ 2 & -4 & 1 & 0 \\ 1 & -2 & 2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 4 \end{bmatrix}$$

$$\left[\begin{array}{cccc|c} 1 & -2 & -1 & 3 & 1 \\ 2 & -4 & 1 & 0 & 5 \\ 1 & -2 & 2 & -3 & 4 \end{array}\right] \longrightarrow \left[\begin{array}{cccc|c} 1 & -2 & 0 & 1 & 2 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

The variables that don't correspond to the pivot columns, x_2 and x_4, are called ______________________.

The solutions are ...

(b) Describe all solutions of the ______________________ system

$$\begin{bmatrix} 1 & -2 & -1 & 3 \\ 2 & -4 & 1 & 0 \\ 1 & -2 & 2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cccc|c} 1 & -2 & -1 & 3 & 1 \\ 2 & -4 & 1 & 0 & 5 \\ 1 & -2 & 2 & -3 & 4 \end{array}\right] \longrightarrow \left[\begin{array}{cccc|c} 1 & -2 & 0 & 1 & 2 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

The solutions are ...

(c) Does $A\mathbf{x} = \mathbf{0}$ have a nontrivial solution?

Fact:

(a) A homogeneous linear system has at least one solution $\mathbf{x}{=}\mathbf{0}$, called the ____________________________.

(b) A _________________________ is a ____________________________ satisfying $A\mathbf{x} = \mathbf{0}$.

7 Sec 1.7 Linear independence

Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -2 \\ -4 \\ -2 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$, and $\mathbf{v}_4 = \begin{bmatrix} 3 \\ 0 \\ -3 \end{bmatrix}$.

(a) Determine (using work we've already done) whether the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is linearly dependent.

Fact:
The columns of a matrix M are linearly independent if and only if the matrix equation $M\mathbf{x} = \mathbf{0}$ has ____.

(b) Determine by inspection whether the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is linearly dependent.

8 Sec 1.8 Intro to linear transformations

- A matrix transformation T is a function (or mapping or map)

$$T : \mathbb{R}^n \to \mathbb{R}^m \quad \text{defined by} \quad T(\mathbf{x}) = A\mathbf{x} \quad \text{for all } \mathbf{x} \text{ in } \mathbb{R}^n$$

  where A is an ________________________

- ________________________ is the set of all images $T(\mathbf{x})$.
- Note: Each image $T(\mathbf{x})$ is of the form $A\mathbf{x}$, ie.

Example: Define a mapping $T : \mathbb{R}^2 \to \mathbb{R}^3$ by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 - 3x_2 \\ 3x_1 + 5x_2 \\ -x_1 + 7x_2 \end{bmatrix} \text{ or } x_1 \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} + x_2 \begin{bmatrix} -3 \\ 5 \\ 7 \end{bmatrix}$$

(a) Find the image $T(\mathbf{u})$ of $\mathbf{u} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ under the mapping T.

(b) Is $\mathbf{b} = \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}$ in the range of T? Is $\mathbf{c} = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}$ in the range of T?

Linear transformations

Def: Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a function.

(i) T preserves _______________

if: $T(\mathbf{u} + \mathbf{v}) = T(\mathbf{u}) + T(\mathbf{v})$ for all $\mathbf{u}, \mathbf{v}$ in the domain of T.

(ii) T preserves _____________________________________

if: $T(c\mathbf{u}) = cT(\mathbf{u})$ for all scalars c in $\mathbb{R}$ and all $\mathbf{u}$ in the domain of T.

The function T is called __________

if:

Sec 1.8 Exercise 41

Show that the function $T : \mathbb{R}^2 \to \mathbb{R}^3$ defined by

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2x - 3y \\ x + 4 \\ 5y \end{bmatrix}$$

is linear or not linear.