Before Mon class, review:

* def of tangent line from Calc I

* Integration techniques
  - u-substitution
  - integration by parts: $\int u\,dv + \int v\,du = uv$

    $\int u\,dv = uv - \int v\,du$

Reading HW: Sec 14.2

Example 2 (unit tangent vector)

Example 3 (New derivative rules)

Example 5 (Indefinite integral)

14.1 Vector-valued functions

The function $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$

is called a <u>vector-valued function.</u>

Also written $\vec{r}(t) = x(t)\,\hat{i} + y(t)\,\hat{j} + z(t)\,\hat{k}$

Input is a number t, output is a vector that depends on t.

Each of $x(t)$, $y(t)$, $z(t)$ is a function of t

The <u>domain</u> of $\vec{r}$ is the set of possible inputs t for $x(t)$, $y(t)$, and $z(t)$.

Ex MML #8

Domain of $\vec{r}(t) = \left\langle \sqrt{16-t^2},\ \sqrt{t},\ \dfrac{3}{\sqrt{7+t}} \right\rangle$ is ... ?

$x(t) = \sqrt{16-t^2}$ $\quad y(t) = \sqrt{t}$ $\quad z(t) = \dfrac{3}{\sqrt{7+t}}$

Sol: Domain of $x(t)$ is when $16 - t^2 \geqslant 0 \iff 16 \geqslant t^2$

$\iff 4 \geqslant |t|$

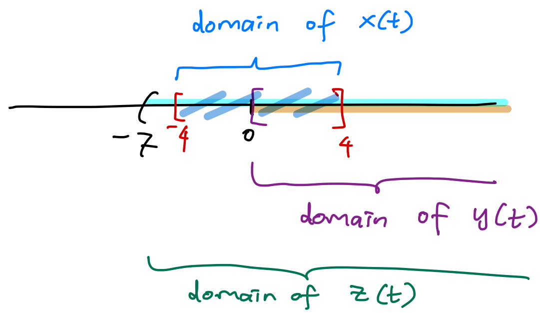
$\iff 4 \geqslant t \geqslant -4$

Domain of $y(t)$ is $\{t : t \geqslant 0\}$

Domain of $z(t)$ is $\{t : 7+t > 0\} = \{t : t > -7\}$

$\therefore$ Domain of $\vec{r}(t)$ is

$\{t : 0 \leq t \leq 4\}$



Ex 5/ MML #9

Let $\vec{r}(t) = \cos(\pi t)\,\hat{\imath} + \sin(\pi t)\,\hat{\jmath} + e^{-t}\,\hat{k}$ for $t \geq 0$.

a) What is $\lim_{t\to 2} \vec{r}(t)$?

$$\lim_{t\to 2} \cos(\pi t) = \cos(2\pi) = 1$$

$$\lim_{t\to 2} \sin(\pi t) = \sin(2\pi) = 0$$

$$\lim_{t\to 2} e^{-t} = e^{-2} = \frac{1}{e^2}$$

$$\lim_{t\to 2} \vec{r}(t) = \left\langle 1, 0, \frac{1}{e^2} \right\rangle = \hat{\imath} + \frac{1}{e^2}\hat{k}$$

Def We say a vector-valued function $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$ is continuous at $t = a$ if each of $f(t)$, $g(t)$, $h(t)$ are continuous at $t = a$

(Recall def $f(t)$ is continuous at $t = a$ if $\lim_{t\to a} f(t) = f(a)$.)

b.)

$\lim_{t\to\infty} \cos(\pi t)$ doesn't exist (DNE)

$\lim_{t\to\infty} e^{-t} = 0$ (exists)

$\lim_{t\to\infty} \vec{r}(t)$ DNE. For this to exist, the limit would have to exist for each component.

Ex 2 Graph the spiral

desmos.com/3D

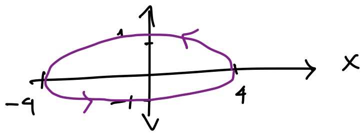
$\vec{r} = \langle 4\cos t, \sin t, \frac{t}{2\pi} \rangle$

where a) $0 \le t \le 2\pi$

b) $-\infty < t < \infty$

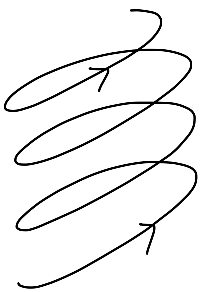
a) Projection of the curve in the xy-plane is

$\langle 4\cos t, \sin t, 0 \rangle$, an ellipse whose positive direction is counter clockwise.



The value of z increases as t gets larger, so the positive direction is up.

b.)



spiral following an elliptical cylinder

Ex 4 Slinky curve

$\vec{r}(t) = (3+\cos 15t)\cos t\ \hat{i}$
$+ (3+\cos 15t)\sin t\ \hat{j}$
$+ \sin 15t\ \hat{k}$

Projection in the xy-plane (set z to 0)
(looking at the curve from above)