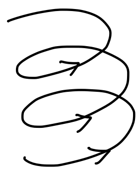
News

Reading HW to do prior to next class:

Textbook Sec 14.1 Vector-valued functions

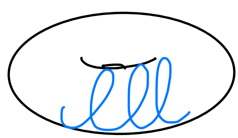
- Example 2: spiral



- Example 3: Roller coaster

  or

  Example 4: slinky curve
  (like spiral but along a donut)



13.6 Cylinders and quadric surfaces

I. Cylinders

(In everyday language, the word "cylinder" describes the shape of a soda can. But in the study of 3D surfaces, "cylinder" describes a more general surface)

Def A <u>cylinder</u> is a surface which is parallel to a line.

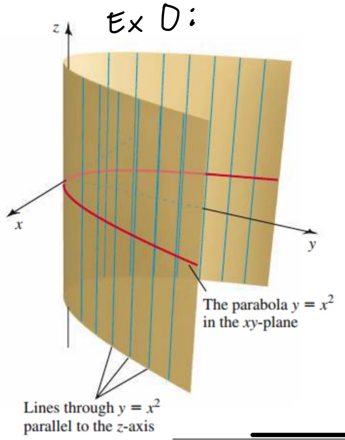
* Most of our examples will be cylinders which are parallel to one of the <u>coordinate axes</u>,
(x-axis, y-axis, or z-axis)

and these can be described by equations in one or two vars,

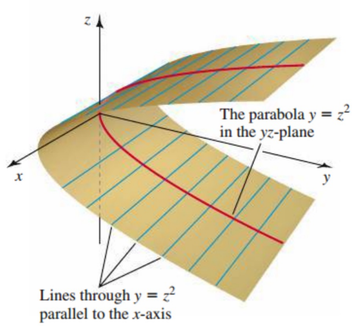
e.g. $x - \sin z = 0$ (parallel to the y-axis)

or $x^2 + 4y^2 = 16$ (parallel to the z-axis)

Ex 0:



Another cylinder example

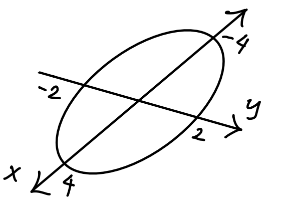


Because z is not mentioned in equation $y = x^2$, z can be any number, so $y = x^2$ describes the cylinder consisting of all lines parallel to the z-axis that pass through the parabola $y = x^2$ in the xy-plane.

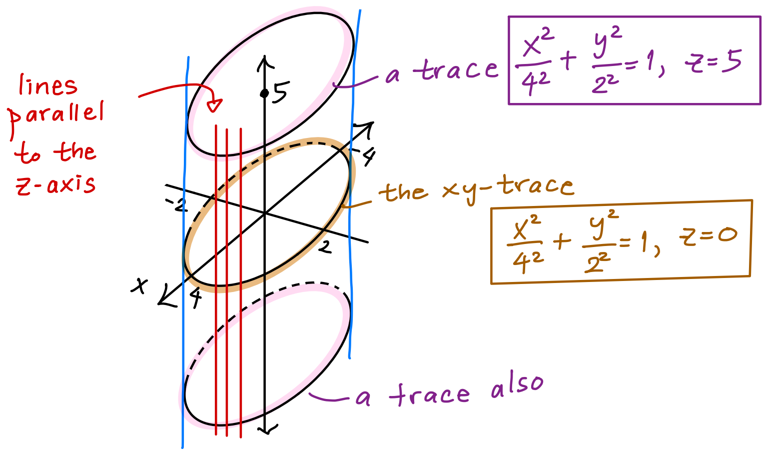
Ex1 (a): The equation $\frac{x^2}{4^2}+\frac{y^2}{2^2}=1$ describes the cylinder consisting of all lines parallel to the z-axis passing through the curve $\underbrace{\frac{x^2}{4^2}+\frac{y^2}{2^2}=1}_{\text{ellipse}}$ on the xy-plane.

because z is not mentioned in eq

Sketch of ellipse on the xy-plane:



Sketch of surface:



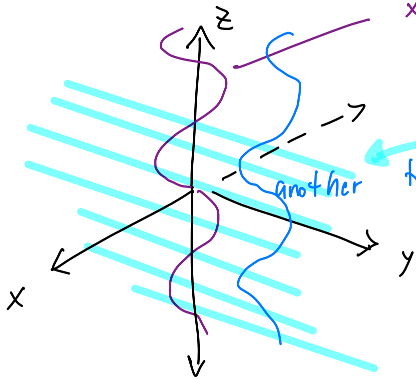
<u>Def</u> * The <u>xy-trace</u> (yz-trace) (xz-trace) of a surface is the set of points at which the surface intersects the xy-plane. (yz-plane) (xz-plane)

— To find an equation for the xy-trace, (yz-trace) (xz-trace) set z (x) (y) to 0

* In general, a <u>trace</u> is when the surface intersects a plane parallel to one of these 3 coordinate planes. (Traces can help us visualize a surface)

Note: This surface  is called an <u>elliptic cylinder</u> because one of the traces is an ellipse.

Ex 1(b) $x - \sin z = 0$ is a cylinder consisting of lines parallel to the y-axis passing through the curve $x = \sin z$ in the xz-plane



xz-trace is $x - \sin z = 0$

trace

Always sketch at least two traces

II. Quadratic surfaces

A <u>quadric surface</u> is described by a 2nd degree equation in 3 variables

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$$

where the coefficients $A, B, \ldots, J$ are constants and at least one of A, B, C, D, E, F is nonzero.

* We'll look at the most common quadric surfaces only, e.g. $\frac{x^2}{3^2} + \frac{y^2}{4^2} + \frac{z^2}{5^2} = 1$ and $z = x^2 - \frac{y^2}{4}$

Ex 2 An <u>ellipsoid</u> is defined by $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

(like a sphere but different distances from the center)

e.g: $\frac{x^2}{3^2} + \frac{y^2}{4^2} + \frac{z^2}{5^2} = 1$

<u>Def</u> An <u>intercept</u> of a surface (if it exists) is where the surface intersects a coordinate axis.

* To find the x-intercepts, set $y=z=0$ then solve for x.

e.g: x-intercepts? $y=z=0$ and $\frac{x^2}{3^2} = 1 \Rightarrow y=z=0$ and $x = \pm 3$

So the x-intercepts of this ellipsoid are $(3,0,0)$ and $(-3,0,0)$.

* Similarly, to find the y-intercepts, set $x=z=0$ & solve for y.

" " z-intercepts, set $x=y=0$ & solve for z.

e.g: The y-intercepts are $(0,4,0)$ and $(0,-4,0)$.

The z-intercepts are $(0,0,5)$ and $(0,0,-5)$.

Con't e.g.: $\dfrac{x^2}{3^2} + \dfrac{y^2}{4^2} + \dfrac{z^2}{5^2} = 1$ $(*)$

Q: What is the xy-trace of this ellipsoid?

A: Set $z=0 \Rightarrow z=0$ and $\dfrac{x^2}{9} + \dfrac{y^2}{16} = 1$ (an ellipse)

Q: What are other traces parallel to the xy-plane?

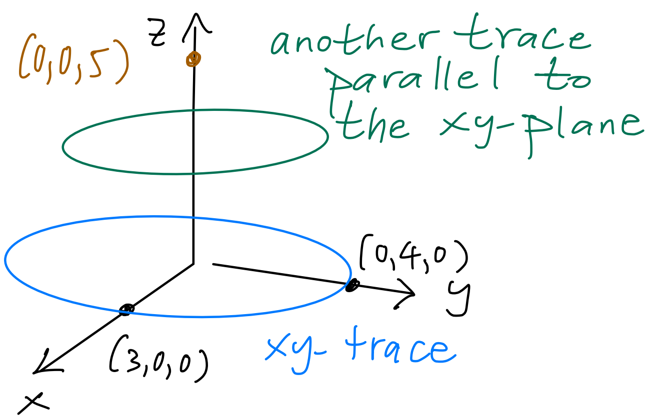
A: Set z to a number $\underbrace{\text{between } -5 \text{ and } 5}$

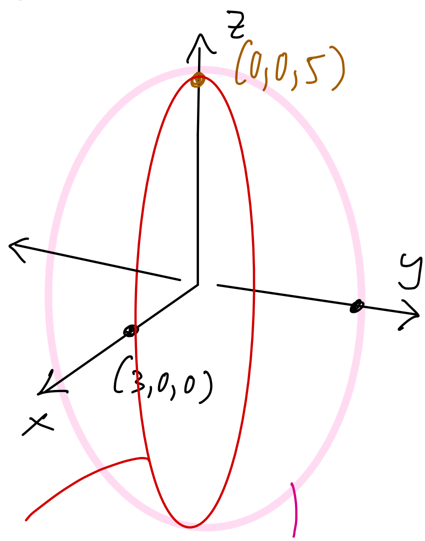
(since RHS of equation $(*)$ is 1)

for example, set $z=1 \Rightarrow \dfrac{x^2}{9} + \dfrac{y^2}{16} + \dfrac{1}{25} = 1$

$$\frac{x^2}{16} + \frac{y^2}{9} = 1 - \frac{1}{25} \quad \text{and } z=1$$

(This is a smaller ellipse than the xy-trace since RHS is less than 1)





xz-trace:

$\dfrac{x^2}{16} + \dfrac{z^2}{25} = 1$ and $y=0$

yz-trace:

$\dfrac{y^2}{9} + \dfrac{z^2}{25} = 1$ and $x=0$

Note: The name ellipsoid is because all traces are ellipses. $\square$

Ex 3 An elliptic paraboloid is defined by $z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$

(or rearrange the variables, ex $x = \frac{y^2}{a^2} + \frac{z^2}{b^2}$)

Ex $z = \frac{x^2}{4^2} + \frac{y^2}{2^2}$

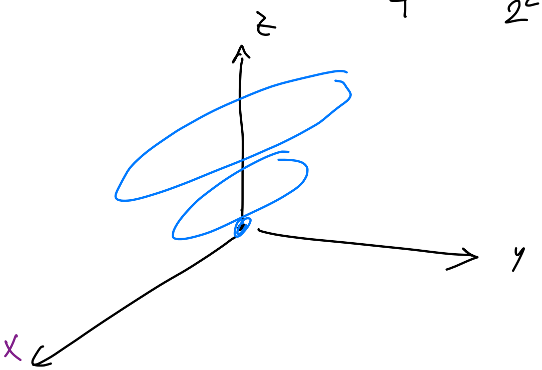
Intercepts? Only $(0,0,0)$

Horizontal trace? • xy-trace: set $z=0 \Rightarrow \frac{x^2}{4^2} + \frac{y^2}{2} = 0$

$\Rightarrow$ point $(0,0,0)$

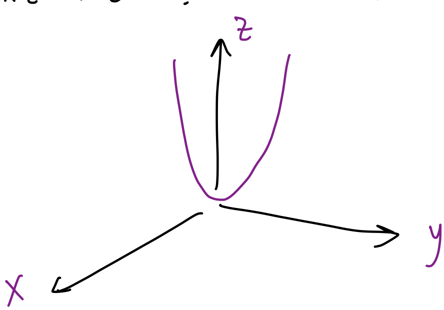
• when $k>0$, $\frac{x^2}{4^2} + \frac{y^2}{2^2} = k$ is an ellipse

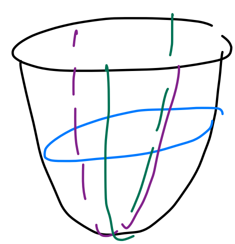
• When $k<0$, $\frac{x^2}{4^2} + \frac{y^2}{2^2} = k$ has no solution



Name elliptic paraboloid is bc traces are ellipses & parabolas

xz-trace? Set $y=0 \Rightarrow z = \frac{x^2}{16}$ (a parabola)





yz-trace? Set $x=0 \Rightarrow z = \frac{y^2}{4}$ (a parabola)

MML # 9

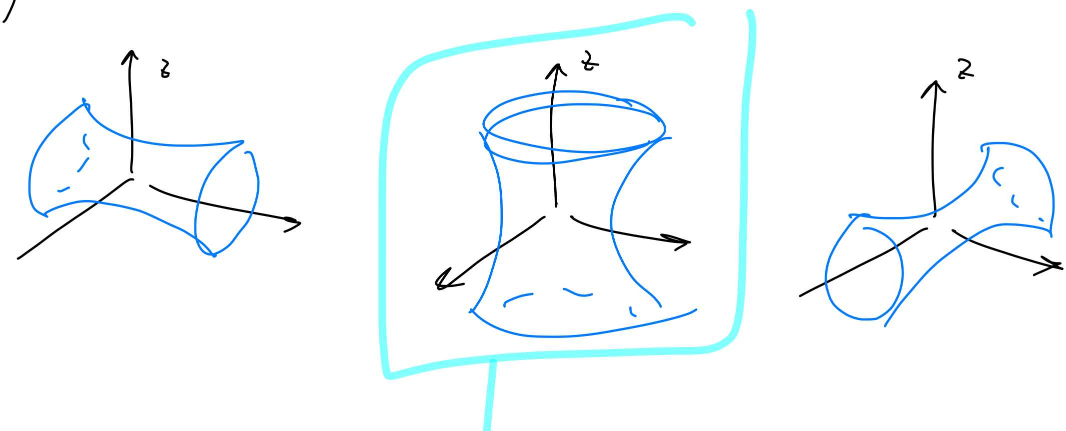
$$\frac{x^2}{100} + \frac{y^2}{100} - \frac{z^2}{100} = 1.$$

a) Find z- intercepts.

Sol: Set $x = y = 0 \implies -\frac{z^2}{100} = 1$

No z-intercepts

c) Choose the correct graph



the only choice where the surface doesn't intersect the z-axis

(Extra Ex)

<u>Ex 5</u> A <u>hyperbolic paraboloid</u> is defined by the eq $z = x^2 - \frac{y^2}{4}$

- xy-trace? $z = 0 \Rightarrow x^2 = \frac{y^2}{4} \Rightarrow$ $y = 2x$, $y = -2x$

Other horizontal trace? Try $z = 1 \Rightarrow 1 = x^2 - \frac{y^2}{4}$

$$\frac{y^2}{4} = x^2 - 1$$

$$y^2 = 4x^2 - 4$$

$$y = \pm\sqrt{4x^2 - 4}$$

hyperbola

- xz-trace? set $y = 0 \Rightarrow z = x^2$ (a parabola)
- yz-trace? Set $x = 0 \Rightarrow z = -\frac{y^2}{4}$ (parabola)

Name hyperbolic paraboloid is bc

hyperbolic ↓ horizontal traces are hyperbolas

paraboloid ↓ two out of 3 traces in coordinate planes are parabolas