What are the basic geometrical objects in 3D?

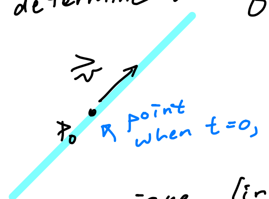
How do we describe them w/ equations?

$\boxed{\text{13.5 Lines \& planes in space}}$

I. Lines in 3D

- Two pts in $\mathbb{R}^3$ determine a unique line (just like how two points in $\mathbb{R}^2$ determine a unique line)
- In $\mathbb{R}^3$, one point and a direction (vector) determine a unique line ℓ

$P_0(x_0, y_0, z_0)$ $\qquad$ $\vec{v} = \langle a, b, c \rangle$



(in $\mathbb{R}^2$, one point & a slope determine a unique line)

- The <u>vector equation</u> for the line ℓ, passing through $P_0(x_0,y_0,z_0)$ in the direction of the vector $\vec{v} = \langle a,b,c\rangle$ is

$$\underbrace{\langle x,y,z\rangle}_{\substack{\text{The position vector}\\ \text{from origin to } P(x,y,z),\\ \text{a point on } \ell}} = \underbrace{\langle x_0,y_0,z_0\rangle}_{\substack{\text{position vector}\\ \text{from origin}\\ \text{to } P_0(x_0,y_0,z_0)}} + t\,\underbrace{\langle a,b,c\rangle}_{\vec{v}} \quad \text{for real numbers } t$$

or write $\vec{r} = \vec{r_0} + t\vec{v}$ for short

- The <u>parametric equations</u> for the same line ℓ is

$$\left.\begin{aligned} x &= x_0 + at \\ y &= y_0 + bt \\ z &= z_0 + ct \end{aligned}\right\} \quad \text{for } -\infty < t < \infty$$

Ex: Ex 2 (book)

Let ℓ be the line through $(-3,5,8)$ and $(4,2,-1)$.

a. Find the vector equation for ℓ

b. Find parametric equations of the line segment from $(-3,5,8)$ to $(4,2,-1)$

Let P_0 $\qquad$ P_1

Sol:

a) Direction of the line is $\vec{v} := \overrightarrow{P_0P_1} = \langle 4-(-3), 2-5, -1-8\rangle$
$= \langle 7,-3,-9\rangle$

Take $\vec{r_0} := \langle -3,5,8\rangle$ (We can also take $\vec{r_0} := \langle 4,2,-1\rangle$ (from P_1))

components of P_0

this operation is scalar multip.

A vector equation is $\underbrace{\langle x,y,z\rangle}_{\vec{r}} = \underbrace{\langle -3,5,8\rangle}_{\vec{r_0}} + t\underbrace{\langle 7,-3,-9\rangle}_{\vec{v}}$

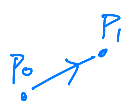
or $\langle x,y,z\rangle = \langle -3,5,8\rangle + \langle 7t,-3t,-9t\rangle$

or $\langle x,y,z\rangle = \langle -3+7t, 5-3t, 8-9t\rangle$

set each component equal

b) Parametric equations for ℓ are

$$\left.\boxed{\begin{aligned} x &= -3+7t \\ y &= 5-3t \\ z &= 8-9t \end{aligned}}\right\} \text{ for } -\infty < t < \infty$$



To find correct interval for t for the line segment, note when $t=0$, $(x,y,z) = (-3,5,8) = P_0$.

Set $(x,y,z) = P_1 = (4,2,-1) \Rightarrow 4 = -3+7t \Rightarrow 7 = 7t \Rightarrow t = 1$

Interval of t should be: $\boxed{0 \le t \le 1}$