First & Last Name: ____________________ Student ID: ____________

Math 2310 Multivariable Calculus III Group Quiz Seven (No notes/phones)

1. Who are the other people in your group? What is their favorite breakfast food? What is your group number?

2. What is an equation of the lower half of the circle in the xy-plane? Sketch them.

3. Write an equation of the lower hemisphere of the sphere of radius 10 centered at the origin. Sketch them.

4. Write down the equation of a paraboloid which opens up, whose z-intercept is $(0, 0, 5)$. Write down the same paraboloid which opens down.

5. Let $f(x, y) = 9y - x^2y - y^2 + 6$.

   (a) Compute ∇f.

   (b) Find a unit vector $\mathbf{v}$ which gives the direction of steepest descent (fastest decrease) for the function $f(x, y)$ at the point $(x, y) = (3, -1)$. Then compute $D_{\mathbf{v}}f(3, -1)$.

   (c) Find a nonzero vector in $\mathbb{R}^2$ which gives a direction of no change in the function $f(x, y)$ from the point $(x, y) = (3, -1)$.

(d) Find the critical points of $f(x, y)$, then classify them using the second derivative test.

(e) **For this part only**, assume that x, y are both functions of t, u. Set up the chain rule for computing $\frac{\partial f}{\partial t}$. Then compute $\frac{\partial f}{\partial t}$ if $x = t^3 u$ and $y = tu^2$.

6. Let R be the region in the xy-plane bound by the lines $x + y = 2$, $y - x = 2$, and $y = 5$. Set up a single iterated double integral to compute $\iint_R f(x, y)\, dA$ using the order of integration $dx\, dy$ or $dy\, dx$ (whichever one is appropriate). Then compute the double integral using the function $f(x, y) = x^2$.

Answers:

1. Group members
2. $y = -\sqrt{r^2 - x^2}$ (lower semicircle)
3. Sphere of radius 10 centered at the origin: $x^2 + y^2 + z^2 = 100$
   Solving for z, we get $z = \sqrt{100 - x^2 - y^2}$ (hemisphere above the xy-plane) and $\boxed{z = -\sqrt{100 - x^2 - y^2}}$ (hemisphere below the xy-plane).
4. Basic circular paraboloid: $z = x^2 + y^2 + 5$ (opens up) and $z = 5 - x^2 - y^2$ (opens down).
5. (a) $\langle -2xy, 9 - x^2 - 2y \rangle$
   (b) $\mathbf{v} = \langle -\frac{3}{\sqrt{10}}, -\frac{1}{\sqrt{10}} \rangle$,
   $D_{\mathbf{v}} f(3, -1) = -2\sqrt{10}$
   (c) $\langle 1, -3 \rangle$ or any nonzero scalar multiple of this vector
   (d) Critical points: $(0, \frac{9}{2})$, $(3, 0)$, $(-3, 0)$
   $(0, \frac{9}{2})$ is a local maximum, and the other two points are saddle points.
   (e) Chain rule: $\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t}$
   2nd answer: $-7t^6u^4 - 2tu^4 + 9u^2$
6. Iterated integral: $\int_2^5 \int_{-y+2}^{y-2} f(x, y)\; dx\; dy$
   2nd answer: $\iint_R x^2\, dA = \frac{27}{2}$