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Math 2310 Multivariable Calculus III Group Quiz Four (No notes/phones)

1. Consider the plane R which contains the line $\mathbf{r}(t) = \langle 1+3t, 2t, 7-5t\rangle$ and the point $P(3,1,4)$.

   (a) Find a normal vector to the plane R.

   (b) Find an equation for the plane R.

2. Consider the line ℓ in $\mathbb{R}^3$ which is parallel to the vector $\mathbf{v} = \langle 2, -2, 1\rangle$ and which contains the point $P(5,0,-3)$. Also, let Q be the point $(6,3,2)$, and let $\mathbf{u} = \overrightarrow{PQ}$.

   (a) Find the equation of the plane which is parallel to the yz-plane and contains the point P.

   (b) Compute the components of the vector $\mathbf{u}$. Express your answer in the form $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$.

   (c) Find an equation for the sphere S which contains the point Q and has center P.

   (d) Write inequalities representing the set of points outside of the sphere S from part (a)

   (e) Find a parametrization of the line ℓ using the parameter t so that the point P corresponds to $t = 0$. Express your answer as a vector function $\mathbf{r}(t) = \langle f(t), g(t), h(t)\rangle$.

   (f) Compute the unit vector in the direction of $\mathbf{v}$.

   (g) Compute the vector in the direction opposite to $\mathbf{v}$ of length 7.

3. Let $\mathbf{r}(t) = e^{3t}(\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})$. Compute the following:

(a) $\int \mathbf{r}(t)\, dt$

(b) $\mathbf{r}'(t)$

4. Consider the vector function $\mathbf{r}(t) = \langle 2t, t^2, \frac{t^3}{3} \rangle$.

(a) Find the velocity and speed of the function $\mathbf{r}(t)$.

(b) Compute the unit tangent vector $\mathbf{T}(t)$.

(c) Compute the curvature at $t = 1$. You can use the curvature formula $\frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|}$ or $\frac{|\mathbf{v} \times \mathbf{a}|}{|\mathbf{v}|^3}$.

(d) Find all points of intersection between the curve $\mathbf{r}(t)$ and the plane $4x - 6y + 3z = 0$.

Answers:

1. (a) A vector which is parallel to the line is $\langle 3, 2, -5 \rangle$. Pick a point Q on the line, for example $Q(1, 0, 7)$. Your answer could be the cross product $\overrightarrow{QP} \times \langle 3, 2, -5 \rangle = \langle 2, 1, -3 \rangle \times \langle 3, 2, -5 \rangle = \boxed{\langle 1, 1, 1 \rangle}$, or any nonzero scalar multiple such as $\boxed{\langle -3, -3, -3 \rangle}$.

   (b) $(x - 3) + (y - 1) + (z - 4) = 0$ or $x + y + z = 8$. Equivalently, any nonzero multiple of this equation such as $5x + 5y + 5z = 40$ would work.

2. (a) $x = 5$

   (b) $\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$

   (c) $(x - 5)^2 + y^2 + (z + 3)^2 = 35$

   (d) $(x - 5)^2 + y^2 + (z + 3)^2 > 35$

   (e) $\mathbf{r}(t) = \langle 5 + 2t, -2t, -3 + t \rangle$ for $t \in (-\infty, \infty)$

   (f) $\frac{2}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{1}{3}\mathbf{k}$

   (g) $-\frac{14}{3}\mathbf{i} + \frac{14}{3}\mathbf{j} - \frac{7}{3}\mathbf{k}$

3. (a) $\frac{1}{3}e^{3t}\mathbf{i} + \frac{2}{3}e^{3t}\mathbf{j} - e^{3t}\mathbf{k} + \mathbf{C}$ where $\mathbf{C} = C_1\mathbf{i} + C_2\mathbf{j} + C_3\mathbf{k}$ is an arbitrary constant vector.

   (b) $3e^{3t}\mathbf{i} + 6e^{3t}\mathbf{j} - 9e^{3t}\mathbf{k}$

4. (a) velocity $\mathbf{v}(t) = \langle 2, 2t, t^2 \rangle$
   speed $|\mathbf{v}(t)| = t^2 + 2$

   (b) $\langle \frac{2}{t^2+2}, \frac{2t}{t^2+2}, \frac{t^2}{t^2+2} \rangle$

   (c) $2/9$

   (d) $(0, 0, 0), (4, 4, \frac{8}{3}), (8, 16, \frac{64}{3})$