

So far we have learned 4 methods for testing convergence/divergence of a SERIES:

- **Geometric Series:** a geometric series is convergent if and only if the ratio is between -1 and 1.
- **Divergence Test:** if the sequence does not converge to 0, the series diverges.
- **Harmonic Series:** the harmonic series diverges.
- **Telescoping Series:** find a formula for the sequence of partial sums, then determine the behavior of the sequence partial sums.

Verify your answer with WolframAlpha:

<https://www.wolframalpha.com/examples/math/calculus/sequences/>

Determine whether each series is convergent or divergent using one of the above four methods (at least 1 of the methods can give you a conclusive answer). State clearly which method/s you apply. **If the series is convergent, find its sum.**

1) $3 - 4 + \frac{16}{3} - \frac{64}{9} + \dots$

2) $2 + 0.5 + 0.125 + 0.03125 + \dots$

3) $\sum_{k=1}^{\infty} \frac{10^k}{(-9)^{k-1}}$

4) $\sum_{k=1}^{\infty} \frac{(-3)^{k+1}}{4^k}$

5) $\sum_{k=0}^{\infty} (-\pi)^k e^{-k}$

6) $\sum_{k=2}^{\infty} \frac{2^{3k+1}}{3^{2k-1}}$

7) $\sum_{k=1}^{\infty} \ln\left(\frac{k+1}{k}\right)$

8) $\sum_{k=1}^{\infty} \frac{1}{k(k+1)}$

9) Let $a_n = \frac{n}{2n+1}$. Use one of the above four methods to determine whether the series $\sum_{n=1}^{\infty} a_n$ is

convergent or divergent. If the series is convergent, compute its sum.

$$10) \frac{1}{3} + \frac{1}{6} + \frac{1}{9} + \frac{1}{12} + \frac{1}{15} + \dots$$

$$11) \frac{1}{3} + \frac{2}{9} + \frac{1}{27} + \frac{2}{81} + \frac{1}{243} + \frac{2}{729} + \dots$$

$$12) \sum_{n=1}^{\infty} \frac{n(n+2)}{(n+3)^2}$$

$$13) \sum_{n=1}^{\infty} \frac{2^n + 3^n}{5^n}$$

$$14) \sum_{n=1}^{\infty} \sqrt[n]{2}$$

$$15) \sum_{n=1}^{\infty} \ln\left(\frac{n+1}{2n+1}\right)$$

$$16) \sum_{n=1}^{\infty} \frac{1}{1 + \left(\frac{2}{3}\right)^n}$$

$$17) \sum_{n=1}^{\infty} (\cos 1)^n$$

$$18) \sum_{n=1}^{\infty} \arctan n$$

$$19) \sum_{n=1}^{\infty} \left(\frac{3}{5^n} + \frac{2}{n}\right)$$

$$20) \sum_{n=1}^{\infty} \frac{2^n}{n^2}$$

$$21) \sum_{n=1}^{\infty} n \sin\left(\frac{1}{n}\right)$$

$$22) \sum_{n=1}^{\infty} \left[\frac{5}{n(n+1)} - (-1)^n \frac{3}{2^n} \right]$$