Math 1152Q: Spring '18 **Week 3 SAMPLE Quiz, Monday, Jan 29**

Topics: Sec 11.1: Definition of convergence, ϵ, N proof, bounded sequences, decreasing/increasing sequences, Monotone Sequence Theorem. Sec 11.2: convergence/divergence of geometric series.

1. (Vocabulary)
   (a) Let $\{b_n\}$ be a sequence. What does it mean to write $\lim_{n\to\infty} b_n = \infty$? Use Def 5, with M and N. Warning: do not include variations of the words "converge", "diverge", "approach", "increase", "continuously", or "infinity" in your answer.
   (Answer: Sec 11.1 Def 5 page 697)

   (b) Given a sequence $\{a_n\}$, what does $\lim_{n\to\infty} a_n = 4$ mean? Use the ϵ and N definition. Do *not* write 'limit', 'converge', 'approach', 'close to 4', etc.
   (Answer: Sec 11.1 Def 2 pg 696)

2. (ϵ, N proof) Let ϵ be a positive number smaller than 1.
   (a) The sequence $a_n = \dfrac{5n^2-9}{n^2-4}$ converges to 5. Choose N so that $|a_n - 5| < \epsilon$ whenever $n > N$.
   (Answer: https://egunawan.github.io/spring18/notes/notes11_1choosingN.pdf)

   (b) The sequence $a_n = \dfrac{n-1}{7n+4}$ converges to 1/7. Choose N so that $|a_n - 1/7| < \epsilon$ whenever $n > N$. Show that this N works.

   (c) Give *a* positive number N such that, $\dfrac{1}{n^2-8} < \epsilon$ for all $n > N$. Show that this N works.

   (d) The sequence $a_n = \dfrac{2n+4}{5n-8}$ converges to 2/5. For any (small) number $\epsilon > 0$, choose N so that if $n > N$, then $\left|\dfrac{2}{5} - a_n\right| < \epsilon$.

   (e) The sequence $a_n = \dfrac{n^2+1}{7n^2+5}$ converges to 1/7. For any (small) number $\epsilon > 0$, find N so that $|1/7 - a_n| < \epsilon$ as long as $n > N$.

3. i.) Fill in the blanks with either the sign $\leq$ or $\geq$.

$$\frac{5n!}{2^n} \ \_\_\_\_\_\_ \ \left(\frac{1}{2}\right)^n \quad \text{for all } n \geq 1$$
$$\frac{n-1}{7n+4} \ \_\_\_\_\_\_ \ \frac{1}{7} \quad \text{for all } n \geq 1$$
$$\frac{n+1}{7n-4} \ \_\_\_\_\_\_ \ \frac{1}{7} \quad \text{for all } n \geq 1$$

   ii.) (Graphing Review) Sketch each function. Label the asymptote/s and zero/s of the graph.
   (a) $f(x) = \dfrac{x-1}{7x+4}$
   Answer:
   - $f(x) = \dfrac{x-1}{7x+4}$ has a horizontal asymptote at $\frac{1}{7}$ because $\lim_{x\to\infty} f(x) = 1/7$.
   - Note that $f(x) = \dfrac{x-1}{7x+4}$ is not defined for $x = -4/7$ and that $-4/7$ is not a zero of the numerator $x-1$. This tells us that $f(x) = \dfrac{x-1}{7x+4}$ has a vertical asymptote at $x = -\dfrac{4}{7}$.
   - Definition: The graph of $y = f(x)$ is said to have a vertical asymptote $x = a$ if $\lim_{x\to a^-} f(x) = \pm\infty$ or $\lim_{x\to a^+} f(x) = \pm\infty$.
   - To figure out whether your graph approaches $+\infty$ or $-\infty$ to the right, plug in a number bigger than a and estimate whether it looks very large (positive) or very large (negative).

(b) $g(x) = \dfrac{x+1}{7x-4}$

(c) $h(x) = \dfrac{1}{x+5}$

iii.) By just looking at your sketches above, determine whether each of the following sequences is increasing or decreasing (or neither) for $n = 1, 2, 3, \ldots$.

(a) $\left\{\dfrac{n-1}{7n+4}\right\}_{n=1,2,3,\ldots}$ (b) $\left\{\dfrac{n+1}{7n-4}\right\}_{n=1}^{\infty}$ (c) $\left\{\dfrac{1}{n+5}\right\}_{n=1}^{\infty}$

iv.) Use your work above to quickly give a lower bound (a number m) and an upper bound (a number M) for each of the following sequences.

(a) $\left\{\dfrac{n-1}{7n+4}\right\}_{n=1,2,3\ldots}$ (b) $\left\{\dfrac{n+1}{7n-4}\right\}_{n=1}^{\infty}$ (c) $\left\{\dfrac{1}{n+5}\right\}_{n=1}^{\infty}$

4. (Sec 11.1 monotone convergence theorem) Recall that lower and upper bounds are not unique!

(a) True or false? The sequence

$$\left\{\frac{3n-6}{6n+2}\right\}_{n=1}^{\infty}$$

is bounded. Give *an* upper bound and *a* lower bound (if true) or justify (if false).

(Answer: True. The sequence is bounded by $-\frac{3}{8}$ and $\frac{1}{2}$ WebAssign Sec 11.1 no. 7, or sketch the corresponding function).

(b) True or false? The sequence

$$\left\{5ne^{-6n}\right\}_{n=1}^{\infty}$$

is bounded. Give *an* upper bound and *a* lower bound (if true) or justify (if false).

(Answer: True. The least upper bound is $\frac{5}{e^6}$ and the greatest lower bound is 0. See WebAssign no. 8, or compute the derivative of the corresponding function to show that this sequence is decreasing.)

(c) True or false? Every bounded sequence is convergent. Justify (if T) or give a counterexample (if F).

(d) True or false? There exists an increasing sequence that converges to 10. Provide an example (if T) or justify (if F).

(e) True or false? There exists an increasing and bounded sequence that does not converge. Provide an example (if T) or justify (if F).

(f) True or false? There is a non-monotonic sequence that converges to 4. Provide an example (if T) or justify (if F).

5. (From WebAssign homework Sec 11.2)

(a) Suppose you're given a mystery sequence whose n-th *partial sum* is known to be $S_n = 4 - 7(3/10)^n$. Calculate the sum of the series $\displaystyle\sum_{n=1}^{\infty} a_n$.

(Answer: 4.)

(b) You'll be given, $\boxed{\text{if } |r| < 1, \text{ we have } \displaystyle\sum_{n=1} r^{n-1} = \frac{1}{1-r}}$. Find ratio values x such that the corresponding geometric series converge. Then compute the sum of the series (assuming x satisfies the condition).

1. $\sum_{n=1}^{\infty}(-4)^n x^n$.
2. $\sum_{n=1}^{\infty}(-4)^{n-1} x^n$.
3. $\sum_{n=1}^{\infty} 5(-4)^{n-1} x^n$.
4. $\sum_{n=0}^{\infty} 4^{n-1} x^n$.

Answer: For each, the interval is $\left(-\frac{1}{4}, \frac{1}{4}\right)$. Don't forget to compute the sum of the series! Check your answers with WolframAlpha.