Names: ____ **Math 1152Q: Spring '18**
Week 11 Sample Quiz

Summary: 11.8: using geometric series test or ratio test to find the interval of convergence of a series; 11.9: find a power series representation of a function and determining the radius of convergence.

0.1 Section 11.8 power series

1. What is a power series?

2. What is the radius of convergence of a power series? What are the different possibilities?

3. In most cases, how do you find the radius of convergence of a power series?

4. From textbook: Find the radius of convergence and interval of convergence of the following series

$$\text{(a.)} \sum_{n=0}^{\infty} \frac{n(x+2)^n}{3^{n+1}}. \quad \text{(b.)} \sum_{n=0}^{\infty} n!x^{2n}. \quad \text{(c.)} \sum_{n=0}^{\infty} \frac{(x-3)^n}{n^5}. \quad \text{(d.)} \sum_{n=0}^{\infty} \frac{(x-3)^n}{n!}.$$

5. From WebAssign: Find the radius R and interval I of convergence of each series.

$$\text{(A.)} \sum_{n=1}^{\infty} \frac{x^n}{6n-1}. \quad \text{(B.)} \sum_{n=1}^{\infty} \frac{6^n(x+7)^n}{\sqrt{n}}. \quad \text{(C.)} \sum_{n=1}^{\infty} (-1)^n \frac{x^{n+1}}{n+4}. \quad \text{(D.)} \sum_{n=0}^{\infty} (-1)^n \frac{x^{5n}}{(2n)!}.$$

11.9 WebAssign using geometric series

For each function, find a power series representation and determine the *interval* of convergence.
(You can check your work with WolframAlpha. Type "series representation of ...")

6. $f(x) = \dfrac{1}{3+x}$

7. $f(x) = \dfrac{x^3}{5+x}$

8. $f(x) = \dfrac{x}{1+10x^2}$

11.9 WebAssign using differentiation and integration of power series

For each function, find a power series representation. Determine the *radius* of convergence.

9. $f(x) = \dfrac{1}{(2+x)^2}$

10. $f(x) = \dfrac{1}{(2+x)^3}$

11. $f(x) = \dfrac{x}{(2+x)^3}$

12. $f(x) = \ln(1+x)$

13. $f(x) = arctan(x)$

14. $\displaystyle\int \frac{1}{1+x^7}\,\text{dx}$

15. $\displaystyle\int \frac{x}{1-x^7}\,\text{dx}$

Solution 1. See Sec 11.8, top of page 747

Solution 2. There are three cases. See Sec 11.8, top of page 749

Solution 3. See the test used in Examples 1-5 in Sec 11.8, pg 747-750: use geometric series test or ratio test.

Solution 4. (a.) See 11.8 Example 5, pg 750. (b.) Sec 11.8 Example 1, pg 747. (c.) Same radius of convergence as 11.8 Example 2, pg 747, but both endpoints are included. (d.) Same answer as 11.8 Example 3, pg 748.

Solution 5. (A.) $R = 1$, $I = [-1, 1)$. (B.) $R = 1/6$, $I = [-43/6, -41/6)$. (C.) $R = 1$, $I = (-1, 1]$. (D.) $R = \infty$, $I = (-\infty, \infty)$

Solution 6. Interval of convergence is $\boxed{(-3, 3)}$. See Sec 11.9 Example 2.

Solution 7. See Sec 11.9 Example 3.

Solution 8. Interval of convergence is $\boxed{(-\frac{1}{\sqrt{10}}, \frac{1}{\sqrt{10}})}$. Use the same strategy as Sec 11.9 Example 3.

Solution 9. Similar to Sec 11.9 Example 5, use differentiation. The power series representation is $\boxed{\sum_{n=0}^{\infty} (-1)^n \frac{(n+1)}{2^{n+2}} x^n}$. The radius of convergence is $\boxed{2}$.

Solution 10. Similar to Sec 11.9 Example 5. Use the answer to question 9 to get the power series representation $\boxed{\sum_{n=0}^{\infty} (-1)^n \frac{(n+1)(n+2)}{2(2^{n+3})} x^n}$. The radius of convergence is the same, $\boxed{2}$.

Solution 11. Similar to Sec 11.9 Example 5. This is just x^2 times the answer to question 10. The power series representation is $\boxed{\sum_{n=2}^{\infty} (-1)^n \frac{n(n-1)}{2(2^{n+1})} x^n}$. The radius of convergence is the same, $\boxed{2}$.

Solution 12. (see Sec 11.9, Example 6)

Solution 13. (see Sec 11.9, Example 7)

Solution 14. (see Sec 11.9, Example 8)

Solution 15. Similar to Sec 11.9, Example 8. Series representation is $\boxed{\sum_{n=0}^{\infty} \frac{x^{7n+2}}{7n+2} + C}$. Radius of convergence is $\boxed{1}$.