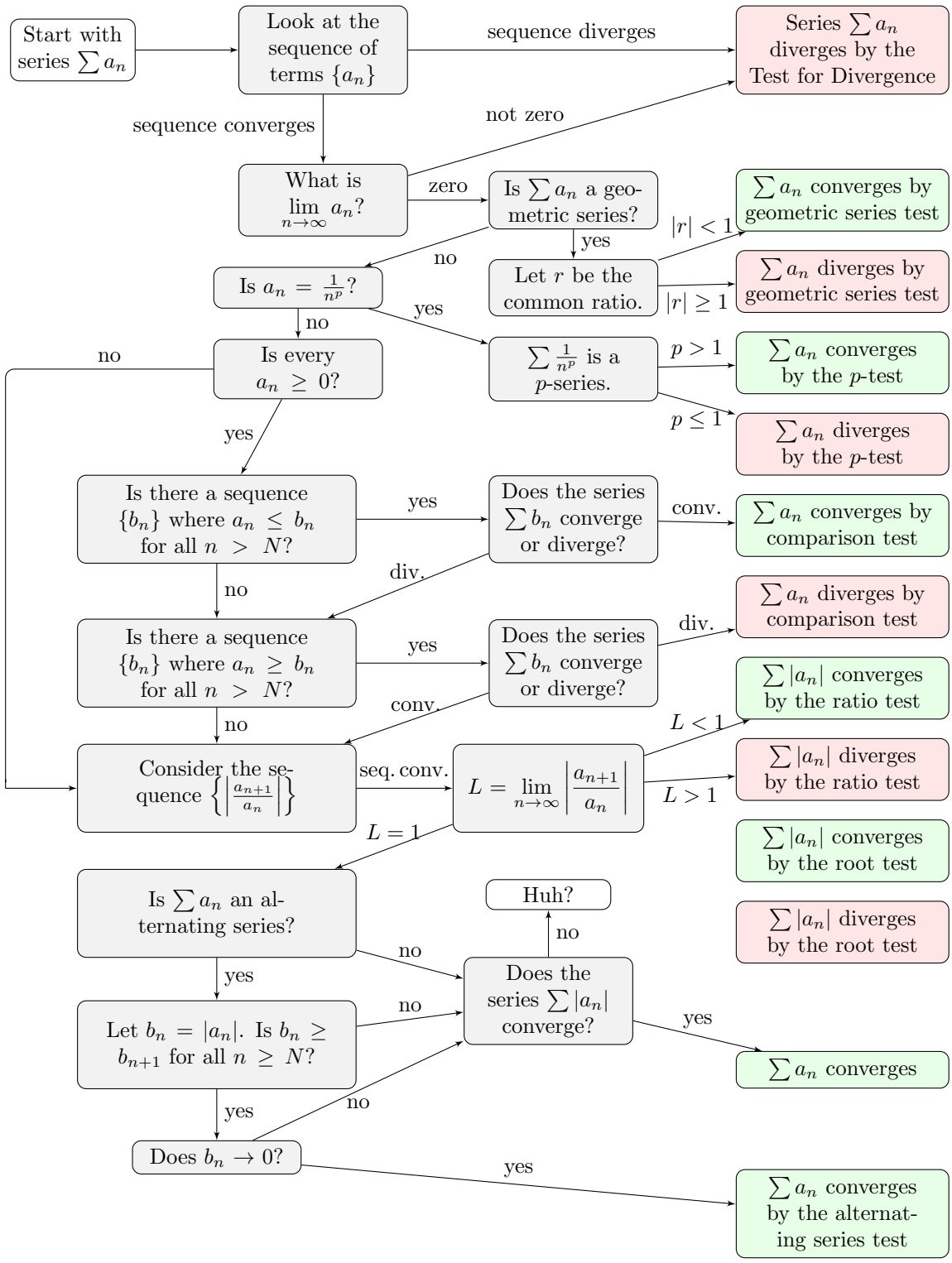
Start with series $\sum a_n$
Look at the sequence of terms $\{a_n\}$
sequence diverges
Series $\sum a_n$ diverges by the Test for Divergence
sequence converges
not zero
What is $\lim_{n\to\infty} a_n$?
zero
Is $\sum a_n$ a geometric series?
yes
no
Let r be the common ratio.
$|r| < 1$
$\sum a_n$ converges by geometric series test
$|r| \geq 1$
$\sum a_n$ diverges by geometric series test
Is $a_n = \frac{1}{n^p}$?
yes
no
$\sum \frac{1}{n^p}$ is a p-series.
$p > 1$
$\sum a_n$ converges by the p-test
$p \leq 1$
$\sum a_n$ diverges by the p-test
Is every $a_n \geq 0$?
no
yes
Is there a sequence $\{b_n\}$ where $a_n \leq b_n$ for all $n > N$?
yes
Does the series $\sum b_n$ converge or diverge?
conv.
$\sum a_n$ converges by comparison test
div.
no
Is there a sequence $\{b_n\}$ where $a_n \geq b_n$ for all $n > N$?
yes
Does the series $\sum b_n$ converge or diverge?
div.
$\sum a_n$ diverges by comparison test
conv.
no
Consider the sequence $\left\{\left|\frac{a_{n+1}}{a_n}\right|\right\}$
seq. conv.
$L = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$
$L < 1$
$\sum |a_n|$ converges by the ratio test
$L > 1$
$\sum |a_n|$ diverges by the ratio test
$L = 1$
$\sum |a_n|$ converges by the root test
$\sum |a_n|$ diverges by the root test
Is $\sum a_n$ an alternating series?
no
yes
Huh?
no
Does the series $\sum |a_n|$ converge?
yes
$\sum a_n$ converges
Let $b_n = |a_n|$. Is $b_n \geq b_{n+1}$ for all $n \geq N$?
no
yes
no
Does $b_n \to 0$?
yes
$\sum a_n$ converges by the alternating series test