 Trigonometric Integrals Name: ______________________

Task 0 fill in the blanks (Copy from Appendix D page A24-A31, Sec 3.3, Sec 7.2)

Trigonometric Identity (Copy from Appendix D A28-A29)

$$\cos^2 x + \sin^2 x = \underline{\hspace{5cm}}.$$

$\cos^2 x = \underline{\hspace{5cm}}$. $\sin^2 x = \underline{\hspace{5cm}}$.

$\sec^2 x = \underline{\hspace{5cm}}$. $\tan^2 x = \underline{\hspace{5cm}}$.

Double Angle Formula (Copy from Sec 7.2 pg 480 or Appendix D pg A29)

$$\cos 2x = \underline{\hspace{5cm}} = \underline{\hspace{5cm}} = \underline{\hspace{5cm}}.$$

Deduce the **Half Angle Formulas (Apdx. D)**:

$\cos^2 x = \dfrac{}{\underline{\hspace{5cm}}}$. $\sin^2 x = \dfrac{}{\underline{\hspace{5cm}}}$.

Derivatives (Copy from Sec 3.3 pg 193)

$\frac{d}{dx}(\sin x) = \underline{\hspace{5cm}}$. $\frac{d}{dx}(\cos x) = \underline{\hspace{5cm}}$.

$\frac{d}{dx}(\tan x) = \underline{\hspace{5cm}}$. $\frac{d}{dx}(\sec x) = \underline{\hspace{5cm}}$.

Useful Anti-Derivatives (Copy from Sec 7.2 pg 482-483)

$\int \tan x \, dx = \dfrac{}{\underline{\hspace{5cm}}}$. $\int \sec x \, dx = \dfrac{}{\underline{\hspace{5cm}}}$.

Task 1 (three parts): Review u-substitution and chain rule/ quotient rule

Review: Evaluate the following indefinite integral using u-substitution with u=cos x.

Anti-Derivative

$$\int \tan x \, dx = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.$$

1a. Check your work or learn how to do this by looking at the book's solution: Sec 5.5 Example 6, pg 415-416. **Write your work here (required).**

Knowing the derivatives for cosine and sine functions, use chain rule/ quotient rule to compute:

Derivative $$\frac{d}{dx}(\tan x) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.$$

1b. Check your work or learn how to do this by looking at the book's solution: Sec 3.3 pg 193. **Write your computation work here (required).**

Given the derivatives for cosine and sine functions, use chain rule or quotient rule to compute

Derivative $$\frac{d}{dx}(\sec x) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.$$

1c. Check your answer or learn how to do it by watching this Khan academy video. (Note: the video uses quotient rule but you may find the chain rule to be less work in this case): https://www.khanacademy.org/math/ap-calculus-ab/ab-derivative-rules/ab-diff-trig-func/v/derivatives-of-secx-and-cscx **Write your work here (required).**

Integrating Products of tangent and secant (power of tangent is odd or power of secant is even) where you can use u-substitution

Let m and n be integers. Evaluate $\int \tan^m x \sec^n x\, dx$.

Strategy: The power of tangent is odd (see Example 6)

$m = 2k + 1 > 0$

$$\begin{aligned}\int \tan^m x \sec^n x\, dx &= \int \left(\tan^2 x\right)^k \sec^{n-1} x \sec x \tan x\, dx \\ &= \int \left(\sec^2 x - 1\right)^k \sec^{n-1} x \sec x \tan x\, dx \\ &= \int \left(u^2 - 1\right)^k u^{n-1}\, du\end{aligned}$$

Strategy: The power of secant is even (see Example 5)

$n = 2k > 0$

$$\begin{aligned}\int \tan^m x \sec^n x\, dx &= \int \tan^m x \left(\sec^2 x\right)^{k-1} \sec^2 x\, dx \\ &= \int \tan^m x \left(1 + \tan^2 x\right)^{k-1} \sec^2 x\, dx \\ &= \int u^m \left(1 + u^2\right)^{k-1}\, du\end{aligned}$$

Examples.

Strategy: The power of tangent is odd (Example 6)

Task 2. Evaluate the indefinite integral given in Sec 7.2, Example 6, pg 482. Write below both the problem and your solution.

Strategy: The power of secant is even (Example 5)

Task 3. Evaluate the indefinite integral given in Sec 7.2, Example 5, pg 481. Write below both the problem and your solution.

Note: If the power of tangent is odd and the power of secant is even, either strategy can be used. But what if neither is true? Will learn a strategy for this later.

Task 4: Evaluate the following using u-substitution. Copy book's solution Sec 7.2 pg 483.

Anti-Derivative

$$\int \sec x \, dx = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ .$$

Write your work here (required):