Selected problems from Exam 1 Practice: Sec 11.2, 11.4, 11.6 **Math 1152Q: Spring '18**

STRATEGY TIPS FOR SERIES WITH ONLY POSITIVE TERMS:

✓ The ratio test usually works efficiently when the term contains factorial like $(n+3)!$ or exponents like 7^n, $\frac{1}{7^n}$.

✗ The ratio test will *not* work with series with ONLY p-series-like terms, for example, $\sum \frac{n^2+4}{\sqrt{n^5-1}}$. Why do you think this is?

✓✗ Only use one of the comparison tests are when the series looks like the geometric series $\sum r^n$, or the p-series $\sum \frac{1}{n^p}$.

Check all the 'does the series [blank] converge' questions below with WolframAlpha.

INSTRUCTION FOR USING the Comparison Test or Limit Comparison Test. For full credit, you should give

i. The series $\sum b_n$ with which you compare and a short statement on why it converges or diverges

ii. An inequality or limit computation

- If using the Comparison Test, give an inequality of the form $a_n \leq b_n$ or $a_n \leq b_n$
- If using the Limit Comparison Test, compute $\lim_{n\to\infty} \frac{a_n}{b_n}$

iii. A conclusion statement.

1. Pages 728-730: Sec 11.4 Examples 1,2,3,4; Pages 740-741: Sec 11.6 Examples 3, 5 (assume all terms are positive).
2. **(Statements of theorem)**
   (a) Write the statement of the *divergence test* as stated in Stewart Sec 11.2 (either box no. 6 or 7 is OK).
   (b) Write the statement of the *comparison test* as stated in Stewart Sec 11.4.
   (c) Write the statement of the *limit comparison test* as stated in Stewart Sec 11.4.
   (d) Write the statement of the *ratio test* as stated in Stewart Sec 11.6
   (e) (You do not need to memorize/use the *root test* but you may use it on a test if you want.)
3. Show whether each series $\sum a_n$ below converges or diverges using the Comparison Test or Limit Comparison Test.
   (a) $\sum_{n=2}^{\infty} \frac{n^3}{n^4-1}$ Hint: divergent, compare with the harmonic series.
   (b) $\sum_{n=1}^{\infty} \frac{6^n}{5^n-1}$ Hint: divergent, compare with a geometric series.
   (c) $\sum_{n=1}^{\infty} \frac{(2n-1)(n^2-1)}{(n+1)(n^2+4)^2}$ Hint: convergent, compare with a p-series.
4. (WebAssign Sec 11.4)
   (a) (no. 2) Determine whether the series $\sum_{n=1}^{\infty} \frac{n}{6n^3+1}$ and $\sum_{n=1}^{\infty} \frac{6}{\sqrt{n^2+5}}$ converge (Hint: ✓compare with a p-series)
   (b) (no. 4) Determine whether the series $\sum_{n=1}^{\infty} \frac{n+6^n}{n+2^n}$ converges or diverges.
   Hints: ✓LCT attempt 1: You try LCT with $\sum(\frac{6}{2})^n$.
   ✓LCT attempt 2: LCT with $\sum \frac{1}{n}$ also works. But this may not be the first thing that comes to your mind.
   ✓Divergence test: the terms are increasing, so this test works.
   ✓Comparison test: find a big enough constant A so that $a_n > A(\frac{6}{2})^n$ - see WebAssign solution.
   ✓Ratio test: you see powers, so you try the ratio test. The ratio $\frac{a_{n+1}}{a_n}$ goes to 6/2.
   (c) (no. 4) Determine whether the series $\sum_{n=1}^{\infty} \frac{2n+3^n}{2n+7^n}$ converges or diverges.
   Hints: ✓LCT attempt 1: You try LCT with $\sum(\frac{3}{7})^n$.
   ✓LCT attempt 2: LCT with $\sum \frac{1}{n^2}$ also works.
   ✗Divergence test: inconclusive.
   ✓Comparison test: find a big enough constant A so that $a_n < A(\frac{3}{7})^n$ - see WebAssign solution.
   ✓Ratio test: you see powers, so you try the ratio test. The ratio $\frac{a_{n+1}}{a_n}$ goes to 3/7.
   (d) (no. 5) Determine whether the series $\sum_{n=1}^{\infty} \frac{5n^2-1}{6n^4+7}$ converges or diverges. (Hint: compare with a p-series and use one of the comparison tests. Would the ratio test be conclusive? See strategy at the top of this file.)
   (e) (no. 6) Determine if $\sum_{n=6}^{\infty} \frac{n-5}{n7^n}$ converges. (Hint: ✓compare with a geometric series. ✓ratio test works bc you see powers $(\frac{1}{7})^n$)
   (f) (no. 6) Determine if $\sum_{n=1}^{\infty} \frac{5^n}{n7^n}$ converges. (Hint: ✓compare with a geometric series. ✓ratio test works bc you see powers $(\frac{5}{7})^n$)
   (g) (no. 6) Determine whether the series $\sum_{n=1}^{\infty} \frac{5^{2n}}{n7^n}$ converges or diverges.
   Hints:
   ✗LCT with geometric series $\sum(\frac{25}{7})^n$ is inconclusive.
   ✓LCT with comparing with the harmonic series $\sum \frac{1}{n}$ works.
   ✓You try the ratio test because you see powers $(\frac{25}{7})^n$.

(h) (no. 7) Determine whether the series $\sum_{n=1}^{\infty} \dfrac{n+8}{n\sqrt{n}}$ converges or diverges.
(Hints:
✓LCT: you compare with a p-series because it looks like one.
✗Ratio test: you try ratio test and it's inconclusive. The top of this file tells you that the ratio test never works for any series that looks ONLY like a p-series.)

5. (Divergence Test Sec 11.2)
   (a) True or false? If a_n does not converge to 0, then the series of $\sum_{n=1}^{\infty} a_n$ diverges.
   (b) True or false? If $\lim_{n\to\infty} a_n = 0$, then the series $\sum_{n=1}^{\infty} a_n$ converges.
   (c) Let $a_n = \frac{4n}{7n+1}$. Determine whether $\{a_n\}$ is convergent. Determine whether $\sum_{n=1}^{\infty} a_n$ is convergent.
   (d) Determine whether the series $\sum_{n=1}^{\infty} \frac{n^2-1}{100+5n^2}$ is convergent or divergent.
   (e) Let $\{a_n\}$ be a sequence which converges to 0. With this information, can you determine whether $\sum_{n=1}^{\infty} a_n$ is convergent? Explain briefly.

6. (WebAssign Sec 11.6: mainly ratio test, but also the divergence test and the comparison tests)
   Determine whether each of the following series $\sum a_n$ converges or diverges.
   (a) $\sum_{n=1} \dfrac{5n!}{2^n}$ (hint: see factorial, think ratio test)
   (b) (no. 2) $\sum_{n=1} \dfrac{n}{5^n}$
   Hints: ✗LCT with geom. series $\sum(\frac{1}{5})^n$ is inconclusive.
   ✓LCT comparing with $\sum \frac{1}{n^2}$ works.
   ✓You see power $(\frac{1}{5})^n$, so you try ratio test.
   (c) (variation of no. 2) $\sum_{n=1} ne^{-5n}$ (see previous hints)
   (d) (no. 3) $\sum_{n=1} \left(\dfrac{1}{4n+1}\right)^n$ Hints:
   ✓LCT: compare with p-series like $\sum \frac{1}{n^2}$.
   ✓LCT: compare with geometric series like $\sum \frac{1}{4^n}$.
   ✓Can use both ratio test and root test because you see powers something^n, but the computation for the ratio test is long.
   (e) (no. 4) $\sum_{n=1} n\left(\dfrac{5}{7}\right)^n$ (hint: ✗see $(\frac{5}{7})^n$, but LCT with geometric series $\sum(\frac{5}{7})^n$ is inconclusive.
   ✓LCT with the p-series $\sum \frac{1}{n^2}$ works.
   ✓try ratio test because you see power $(\frac{5}{7})^n$)
   (f) (no. 5) $\sum_{n=1} \dfrac{|\sin(5n)|}{5^n}$
   (hint: ✓see sin and $(\frac{1}{5})^n$, so think comparison test with the geometric series $\sum(\frac{1}{5})^n$.
   ✗The LCT with $b_n = (\frac{1}{5})^n$ fails.
   ✓The LCT with $b_n = \frac{1}{n^2}$ works.
   ✗Ratio test fails.)
   (g) (variation of no. 5) $\sum_{n=1} \dfrac{|\sin(5n)|}{n^5}$ (Hints: ✓see sin and $(\frac{1}{n^5})$, so think the (non-limit) comparison test with the p-series $\sum(\frac{1}{n^5})$.
   ✗The LCT with $b_n = (\frac{1}{n^5})$ fails.
   ✓The LCT with $\frac{1}{n^3}$ works.
   ✗Ratio test fails.)
   (h) (no. 6) $\sum_{n=1} \dfrac{2^n}{n^3}$. (Hints: ✓Divergence test: numerator grows faster than the denominator, so use divergence test - see WebAssign's solution.
   ✗LCT: you see 2^n, but find that the comparison tests with the geometric series $\sum 2^n$ are inconclusive.
   ✓LCT: you try LCT with $\sum \frac{1}{n}$ and find that it works.
   ✓Ratio test: you can try ratio test because you see power 2^n.
   ✓Root test (not required to memorize): you can try root test because you see power 2^n.)
   (i) (no. 7) $\sum_{n=1} \dfrac{n!}{100^n}$ Hints: ✓Ratio test: you see *factorial* and exponent 100^n, so think ratio test (WebAssign's solution).
   ✓Divergence test: you remember than factorial grows faster than exponential - see sol of WebAssign Sec 11.1 no. 6.
   ✗LCT: You try comparing it with $\sum n!$ but the result is inconclusive.
   (j) (no. 8) $\sum_{n=1} \dfrac{n}{\sqrt{n^3+4}}$ (Hints: ✓LCT or comparison: looks like a p-series, so use either.
   ✗Ratio test: you attempt the ratio test and it doesn't work. But I've told you above that the ratio test will not work for any series that looks like a p-series.)