Name:_________________ **Representations of Functions as Power Series** Notes

A **power series** defines a **function** on its **interval of convergence**.

Task 1. Read pages 752-753. Write a brief summary (1-2 sentences).

Combining Power Series

Theorem Combining Power Series

Suppose the power series $\sum c_n x^n$ and $\sum d_n x^n$ converge absolutely to $f(x)$ and $g(x)$, respectively, on an interval I.

1. **Sum and Difference**

   The power series $\sum (c_n \pm d_n) x^n$ converges absolutely to $f(x) \pm g(x)$ on I.

2. **Multiplication by a power**

   The power series $x^m \sum c_n x^n = \sum c_n x^{n+m}$ converges absolutely to $x^m f(x)$ on I, provided m is an integer such that $k + m \geq 0$ for all terms of the series.

3. **Composition**

   If $h(x) = bx^m$, where m is a positive integer and b is a real number, the power series $\sum c_n \left[h(x)\right]^n$ converges absolutely to the composite function $f(h(x))$ for all x such that $h(x)$ is in I.

Task 2 (Copy Sec 11.9, Example 1 from page 753).
Try to specify where the above Theorem (part 3) is used. What are f (t) and h (x) ?
Give the interval of convergence of the series using above Theorem (part 3).

Task 3 (Copy Sec 11.9 Example 2, pg 753):
Use the geometric series

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \quad \text{for } |x| < 1$$

to find a power series representation for $\dfrac{1}{x+2}$.

Try to specify where the above Theorem (part 3) is used. What are f (t) and h (x) ?
Give the interval of convergence of the new series by applying the above Theorem.

Differentiating and Integrating Power Series

Theorem Differentiating and Integrating Power Series (Sec 11.9, pg 754)

Let the function f be defined by the power series $\sum c_n (x-a)^n$ on its interval of convergence I. THEN:

1. f is a **continuous** function on I.

2. The power series may be differentiated **term by term**.

$$\frac{d}{dx}\left[\sum_{n=0}^{\infty} c_n (x-a)^n\right] = \frac{d}{dx}\left[c_0 + \sum_{n=1}^{\infty} c_n (x-a)^n\right] = \sum_{n=1}^{\infty} c_n n (x-a)^{n-1}.$$

   The resulting power series converges to $f'(x)$ at all points in the interior of I.

3. The power series may be integrated **term by term**.

$$\int\left[\sum_{n=0}^{\infty} c_n (x-a)^n\right] dx = \sum_{n=0}^{\infty} \frac{c_n}{n+1}(x-a)^{n+1} + C \text{ where } C \text{ is a constant.}$$

   The resulting power series converges to $\int f(x)\,dx$ at all points in the interior of I.

Task 4 (Copy the book's solution for Sec 11.9, Example 5, pg 755):
Apply the first theorem and differentiate the geometric series

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \text{ for } |x| < 1$$

to find a series representation for $\dfrac{1}{(1-x)^2}$ and give the interval of convergence of the new series.

Task 4. (Follow Sec 11.9, Example 8a, pg 756.)

Consider the geometric series $\dfrac{1}{1-x}=\sum_{n=0}^{\infty}x^n$ for $|x|<1$.

i. Find a power series representation for $\dfrac{1}{1+x^3}$. (See similar Example 8a, top half).

Try to specify where the first Theorem (part 3) is used. What are f (t) and h (x) ?

ii. Evaluate $\displaystyle\int \frac{1}{1+x^3}\,dx$ as a power series and give the radius of convergence of the new series.

(See the similar solution of Example 8a, bottom half).

iii. Evaluate $\int_0^{0.1} \frac{1}{1+x^3}\,dx$ as a series. (Follow the top-half similar solution of Example 8b).

iv. Use the Alternating Series Remainder Theorem to find a bound on the error in approximating part (c) by adding up the first 4 terms of the series.
(Follow the bottom-half solution of Example 8b, pg 757. Use technology/ calculator).