Remainders in Alternating Series (pg 735)

Theorem **Alternating Series Estimation Theorem**

Let $R_n = S - S_n$ be the **remainder** in approximating the value of a convergent alternating series $\sum_{n=1}^{\infty}(-1)^{n+1} b_n$ by the sum of its first n terms. Then

$$|R_n| \le b_{n+1}.$$

In other words, the remainder is less than or equal to the magnitude of the first neglected term.

Example: Consider following convergent alternating series.

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n} = 1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\cdots \quad \ldots \quad = \ln 2$$ (See Exercise 36 pg 737)

a) Is the sum of sandwiched between any two consecutive partial sums?

b) Is it sandwiched between any two (non-consecutive) partial sums? Sketch the picture from Ex. 1 pg 734).

c) How many terms of $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}$ must be summed to be sure that the remainder is less than 10^{-4}?

d) Find the sum of the series $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}$ correct to 2 decimal places.

<u>Def</u>: A number r is called <u>rational</u> if

<u>Fun Fact</u>: Use the **Alternating Series Estimation Theorem** to prove that e is irrational.

<u>Fact we'll learn later in Sec 11.10</u>: 1/e =