

The Comparison Test

Go to page 727 Sec 11.4. Skim the two paragraphs.

Task 1. Fill in the blanks by copying from the bottom of this page.

Theorem The Comparison Test

Suppose $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are infinite series with **positive** terms.

- If $a_n \leq b_n$ and $\sum_{n=1}^{\infty} b_n$ **converges**, then _____.
- If $a_n \geq b_n$ and $\sum_{n=1}^{\infty} b_n$ **diverges**, then _____.

Task 2. Fill in the blanks below (it's OK if you write 'not sure', but try asking friends or Google first)

- If $a_n \leq b_n$ and $\sum_{n=1}^{\infty} b_n$ **diverges**, then _____.
- If $a_n \geq b_n$ and $\sum_{n=1}^{\infty} b_n$ **converges**, then _____.

Task 3. Example:

Type "sum n/(n^2 + 1), n=1 to infy" into WolframAlpha and write down the Convergence tests that WolframAlpha tried to use (this should appear right after the result):

Task 4a. Don't try to solve yet. We want to use the **Comparison Test** to determine whether the series $\sum_{n=1}^{\infty} \frac{n}{n^2 + 1}$ converges. Below, please compare the terms of this series with the terms of the harmonic series (Sec 11.2 Example 9 on page 713). In particular, write down which series has larger terms and why.

Task 4b. Can you use the Divergence Test from Sec 11.2 to determine whether the series above converges or diverges? Explain.

The Limit Comparison Test

Task 5. First go to page 729 Sec 11.4. Fill in the blanks.

Theorem The Limit Comparison Test

Suppose $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are infinite series with **positive** terms. Let $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = L$.

- If L is a positive *number*, then (fill in the blank by copying the book Sec 11.4 pg 729)

_____.

- If $L = 0$ and $\sum_{n=1}^{\infty} b_n$ **converges**, then _____

(fill in the blank by copying the question of Sec 11.4 p40 on page 732).

- If L is replaced with the infinity symbol and $\sum_{n=1}^{\infty} b_n$ **diverges**, then

_____.

(fill in the blank by copying the question of Sec 11.4 p41 on page 732).

Task 6. Fill in the blanks below (it's OK if you write 'not sure', but try asking friends or Google first)

- If $L = 0$ and $\sum_{n=1}^{\infty} b_n$ **diverges**, then _____.

- If $L = \infty$ and $\sum_{n=1}^{\infty} b_n$ **converges**, then _____.

Task 7. Go to pg 728 Sec 11.4. The paragraph (right before Example 1) tells you how to fill in the blanks.

p-series Test

The p -series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is

convergent if _____ and divergent if _____.

Example: Consider the series the series $\sum_{n=1}^{\infty} \frac{n^4 - 2n^2 + 3}{2n^6 - n + 5}$. We will compare this series with a p -

series $\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{n^p}$ where $p=2$ (this p -series converges by the p -series test above).

Task 8a. Below, compute L where $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = L$.

Task 8a. Using the Limit Comparison Test and your computation above, determine whether the series $\sum_{n=1}^{\infty} \frac{n^4 - 2n^2 + 3}{2n^6 - n + 5}$ converges.

Task 9. Write some questions you have. If you have no questions, write the key skills/concepts you learned from this reading homework.