Telescoping Series

Telelscoping Series Example:

Consider the infinite series $\sum_{k=1}^{\infty}\left[\tan^{-1}(k+1)-\tan^{-1}k\right]$.

a. Find a formula for the n-th term of the sequence of **partial sums** $\{S_n\}$.

b. Evaluate $\lim_{n\to\infty} S_n$ to obtain the value of the series or state that the series diverges.

Telescoping Series Example

Consider the infinite series $\sum_{k=1}^{\infty}\left(\sqrt{k+1}-\sqrt{k}\right)$.

(1) Find a formula for the n-th term of the sequence of **partial sums** $\{S_n\}$.

(2) Evaluate $\lim_{n\to\infty} S_n$ to obtain the value of the series or state that the series diverges.

Telescoping Series Example

Consider the infinite series $\sum_{k=1}^{\infty}\ln\left(\frac{k+1}{k}\right)$.

a. Find a formula for the n-th term of the sequence of **partial sums** $\{S_n\}$.

b. Evaluate $\lim_{n\to\infty} S_n$ to obtain the value of the series or state that the series diverges.

<u>Partial fraction decomposition</u>: (Copy from Sec 7.4 Example 2, pg 494-495)

<u>Partial fraction decomposition and telescoping Series Example</u> (Copy Sec 11.2 Example 8, pg 712)

Consider the infinite series $\sum_{k=1}^{\infty} \frac{1}{k(k+1)}$.

a. Find a formula for the n-th term of the sequence of **partial sums** $\{S_n\}$.

b. Evaluate $\lim_{n\to\infty} S_n$ to obtain the value of the series or state that the series diverges.

Properties of Convergent Series

Theorem (Theorem 8, pg 714)

If $\sum a_k$ and $\sum b_k$ are convergent series, ...

- then the series $\sum ca_k$ **converges** and

$$\sum ca_k = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ .$$

- then the series $\sum (a_k \pm b_k)$ **converges** and

$$\sum (a_k \pm b_k) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ .$$

- if M is a positive integer, then $\sum_{k=1}^{\infty} a_k$ and $\sum_{k=M}^{\infty} a_k$ both converge or both diverge.

Note

Whether a series converges does not depend on a finite number of terms added to or removed from the series. However, the value of a convergent series does change if nonzero terms are added or deleted.

Telescoping Series + Geometric Series + Applying Series Laws Example

Determine whether the series $\sum_{n=1}^{\infty} \left[\frac{5}{n(n+1)} - (-1)^n \frac{3}{2^n} \right]$ is convergent or divergent. If it is convergent, find its sum.