Series

Divergence Test

Task 1: **Theorem** (copy from Theorem 6, pg 713)

If the series $\sum_{k=1}^{\infty} a_k$ is convergent, then $\lim_{k\to\infty} a_k =$ ____________________ .

What does this theorem say? Recall that with any series $\sum a_n$ we associate two sequences:

- the sequence $\{a_n\}$ of its **terms** and
- the sequence $\{S_n\}$ of its **partial sums**.
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If $\sum a_n$ is convergent to S, then

$\lim_{n\to\infty} S_n =$ ____________________ and $\lim_{n\to\infty} a_n =$ ____________________ .

Task 2: Fill in the two blank spaces. Hint 1: see Theorem 6. Hint 2: Copy from NOTE 1, bottom of pg 713.

Proof of Theorem 6:

Task 3: Go to pg 713. Take 10-20 minutes to read and copy the proof on a separate piece of paper. Close the textbook and reproduce the same proof below (opening the textbook as needed).

Task 4: What is the definition of contrapositive? (Please google 'contrapositive definition'. An informal, imprecise explanation is OK).

____.

The following is the contrapositive of Theorem 6:

Task 5: Test for Divergence/ Divergence Test (copy Theorem 7, pg 713)

If $\lim_{k\to\infty} a_k \neq 0$, then the series $\sum_{k=1}^{\infty} a_k$ is ________________________.

Task 6: Copy from NOTE 2 on the bottom of pg 713.

If ________________________, then **the test is inconclusive**. The test cannot be used to determine convergence.

Task 7: Theorem Harmonic Series (an important exampe)

The **harmonic series** $\sum_{k=1}^{\infty} \frac{1}{k} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$ diverges / converges. (Circle one, see Ex 9, pg 713.)

Task 8: Solution of Example 9, copy from pg 713:

Task 9: We have 1/k $\rightarrow$ _________ as k $\rightarrow \infty$. (Copy from Note 2, bottom of pg 713)

Task 10: Copy the instruction and solution for Example 10, pg 714.

Task 11: Example: (Using Example 10 as your guide, solve the following problem.)

Determine whether the series $\sum_{k=1}^{\infty} \frac{k}{2k+1}$ diverges or state that the test you used is inconclusive.

First step: We have k/(2k+1) → _______ as k → ∞.

Second step: The Divergence Test is conclusive/inconclusive.

Task 12: Example (Attempting to use the Divergence Test from Theorem 7):

Determine whether the series $\sum_{k=1}^{\infty} \frac{k}{k^2+1}$ diverges or state that the test you used is inconclusive.

First step: We have $k/(k^2+1)$ → ______ as k → ∞.

Second step: The Divergence Test is conclusive/inconclusive.

Task 13: Write down one or more questions you have related to this reading homework.