Formal definition

Given a sequence $\{a_n\}$, what does $\lim_{n\to\infty} a_n = 2$ mean? Use ϵ and N definition.

Let $\{b_n\}$ be a sequence. What does it mean to write $\lim_{n\to\infty} b_n = \infty$? Use M, N definition.

Warm-up Exercises:

Let $a_n = \dfrac{2n+4}{5n}$ for $n = 1, 2, 3, \ldots$. The sequence $\{a_n\}_{n=1}^{\infty}$ converges to $2/5$.

Let's do a reality check. Let $\epsilon = \dfrac{1}{10}$. Can you find N so that $\left|a_n - \dfrac{2}{5}\right| < \dfrac{1}{10}$ whenever $n > N$?

Scratch work (for yourself):

Polished answer:

Warm-up Exercise:

The sequence $\left\{\dfrac{5}{n^2-8}\right\}_{n=3}^{\infty}$ converges to 0. Reality check: Choose a (positive) number N such that, if $n > N$, then $\left|\dfrac{5}{n^2-8}\right| < \dfrac{1}{100}$.

Scratch work (for yourself):

Polished answer:

Example:

Let $a_n = \dfrac{2n+4}{5n}$ for $n = 1, 2, 3, \ldots$. The sequence $\{a_n\}_{n=1}^{\infty}$ converges to $2/5$.

To show this, let $\epsilon > 0$. Choose N so that $\left| a_n - \dfrac{2}{5} \right| < \epsilon$ whenever $n > N$.

Scratch work (for yourself):

Polished answer:

Example:

The sequence $\left\{ \dfrac{5}{n^2 - 8} \right\}_{n=3}^{\infty}$ converges to 0. Suppose I give you a positive ϵ. (For convenience, assume $\epsilon < 1$). Choose *a* positive number N such that, if $n > N$, then $\left| \dfrac{5}{n^2 - 8} \right| < \epsilon$.

Scratch work (for yourself):

Polished answer: