Polar Coordinates

Motivation

Euler's Formula (using Taylor series) to Polar Coordinates:

https://www.khanacademy.org/math/calculus-home/series-calc/maclaurin-taylor-calc/v/euler-s-formula-and-euler-s-identity

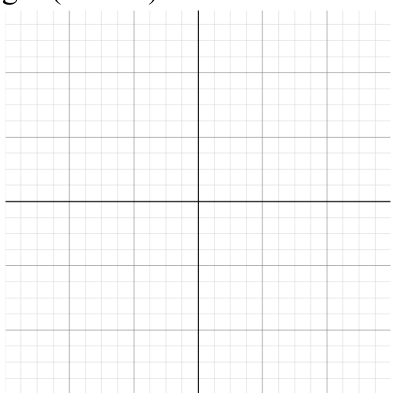
Euler's Formula and Polar Coordinates:

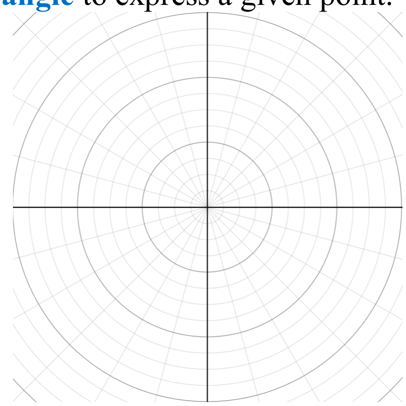
<u>Euler Formula:</u>

<u>Polar Coordinates</u>

(In Multivariable Calculus: polar coordinates $\to$ cylindrical and spherical coordinates in 3D)

Instead of using **horizontal distance** and **vertical distance** from the axes, we use the distance from the origin (**radius**) as well as the corresponding **angle** to express a given point.



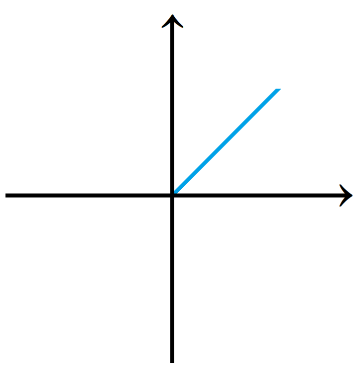


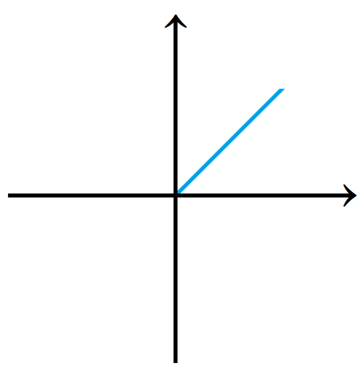
The origin is called the **pole**, and the positive x-axis is called the **polar axis**. The polar coordinates for a point P have the form (r,θ), where the **radial coordinate** r describes the distance from the origin to P, and the **angular coordinate** θ describes an angle starting from the positive x-axis and ending on the ray that passes through the origin and P. As usual, positive angles are measured **counterclockwise** from the positive x-axis.

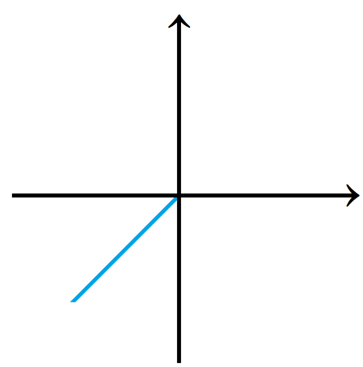
Caution

The representation for a given point is **not unique** in the polar coordinate system.

(r,θ), ________________ and ________________ refer to the same point.







The origin is specified as $(0,\theta)$ in polar coordinates, where θ is any angle.

Example:

Graph the point $\left(1, \frac{5\pi}{4}\right)$ in polar coordinates. Give two alternative representations for the point.

Converting Between Cartesian and Polar Coordinates

Procedure **Converting Coordinates**

A point with polar coordinates (r, θ) has Cartesian coordinates (x, y), where

____ .

A point with Cartesian coordinates (x, y) has polar coordinates (r, θ), where

____ .

Example:

Express the point with polar coordinates $\left(2, \frac{3\pi}{4}\right)$ in Cartesian coordinates.

Example:

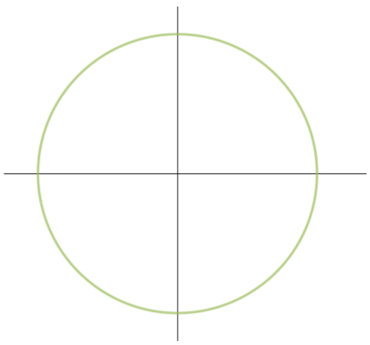
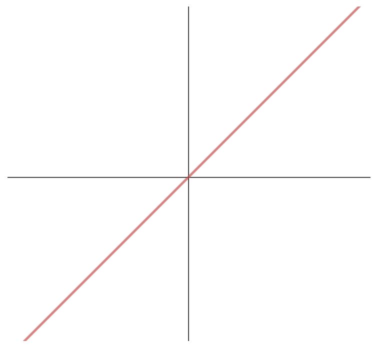
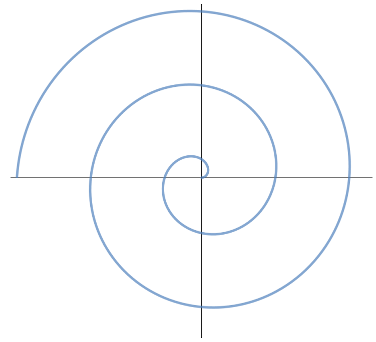
Express the point with Cartesian coordinates $(1, -1)$ in polar coordinates.

Converting Between Cartesian and Polar Equations

A curve in polar coordinates is the set of points that satisfy an equation in r and θ. Some sets of points are easier to describe in polar coordinates than in Cartesian coordinates.

For example,

- The polar equation $r = 3$.
- The polar equation $\theta = \frac{\pi}{4}$.
- The polar equation $r = \theta$.

(Graph with Desmos https://www.desmos.com/calculator/zpm8i8nbsb)

Example:
Convert the polar equation $r\cos\theta = -4$ to a Cartesian equation.

Example:
Convert the polar equation $r = 8\sin\theta$ to a Cartesian equation.

Graphing in Polar Coordinates

Example (Ex 7 pg 662, Fig 10,11):
Graph the polar equation $r = 1 + \sin\theta$.