Tangent Lines

Suppose $h(t)$ and $k(t)$ are differentiable functions of t. Consider the parametric curve

$$\begin{cases} x = h(t) \\ y = k(t) \end{cases}, \text{ where } y \text{ is also a differentiable function of } x.$$

First Derivative

$$\frac{dy}{dx} = \underline{\qquad\qquad} \qquad \text{if } \underline{\qquad\qquad}$$

- The curve has a **horizontal tangent** when ______________________________.
- The curve has a **vertical tangent** when ______________________________.

Example: Consider the parametric curve (**cycloid**) $\Gamma : \begin{cases} x = 2(\theta - \sin\theta) \\ y = 2(1 - \cos\theta) \end{cases}$ where $0 \leq \theta \leq 2\pi$. A *cycloid* is the curve traced out by a point on the circumference of a circle as the circle rolls along a straight line: `https://www.desmos.com/calculator/duk9jpvevu`

(a) Find the slope of the tangent line at the point where $\theta = \frac{\pi}{3}$.

(b) At what point/s is the tangent line horizontal?

(c) At what point/s is the tangent line vertical?

Second Derivative

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \underline{\hspace{4cm}}$$

Example 2 pg 650: Compute the second derivative for Γ whenever it's defined and use that to determine concavity of the curve.

Areas

Example 3 pg 651: Consider the parametric curve (**cycloid**) $\Gamma : \begin{cases} x = 2(\theta - \sin\theta) \\ y = 2(1 - \cos\theta) \end{cases}$ where $0 \leq \theta \leq 2\pi$.

i.) Sketch the area enclosed by Γ and the x-axis.

Area

Suppose that the curve is traced out once by the parametric equations $\begin{cases} x = h(t) \\ y = k(t) \end{cases}$, $\alpha \le t \le \beta$, where $h(t)$ is **monotonic** and $k(t) \ge 0$. Then, by Substitution Rule, the area is

$$A = \int_a^b y\,dx = \underline{\hspace{6cm}}.$$

ii.) Find the area enclosed by Γ and the $x-$axis.

iii.) Conduct a reality check. For example, you can compare your sketch in part (i) and your result in part (ii).