

NAME : \_\_\_\_\_

VERIFY ALL ANSWERS WITH A COMPUTING TOOL (like WolframAlpha) when possible.

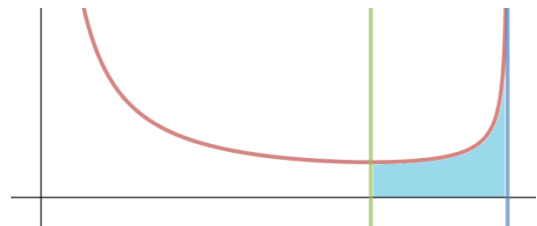
1. For what values of  $p$  is  $\int_1^{\infty} \frac{1}{x^p} dx$  convergent? You don't need to show work (Hint: see Ex 4 pg 530).

2. For what values of  $p$  is the integral  $\int_0^1 \frac{1}{x^p} dx$  convergent?

3. Evaluate  $\int_0^{\infty} xe^{-x} dx$ . Use integration by parts and limit laws.

4. Find the area of the region enclosed by the graph of  $f(x) = \frac{1}{x\sqrt{9-x^2}}$  and the  $x$ -axis on

the interval  $\left[\frac{3\sqrt{2}}{2}, 3\right]$ .



**Improper Integral HW**

5. Let  $R$  be the region bounded by the function  $f(x) = \frac{1}{\sqrt{1-x^2}}$  and  $x$ -axis on the interval  $[0,1]$ . Evaluate the area of  $R$ .

6. Evaluate  $\int_0^1 \frac{\ln x}{\sqrt{x}} dx$ .

**Improper Integral HW**

7. Evaluate  $\int_0^3 \frac{1}{x^2 - 6x + 5} dx$ .

(Hint: Use partial fraction decomposition to write the integrand as the sum of two fractions).

8. Evaluate  $\int_0^{\frac{\pi}{2}} \sec^4 x dx$ .

## Improper Integral HW

9. Evaluate  $\int_0^{\infty} \frac{1}{\sqrt{x}(1+x)} dx$ .

10. If  $f(t)$  is continuous for  $t \geq 0$ , the **Laplace Transform** of  $f$  is the function  $F$  defined by

$$F(s) = \int_0^{\infty} f(t)e^{-st} dt$$

and the domain of  $F$  is {all numbers  $s$  for which the improper integral converges}.  
Your task: for each function below, compute its **Laplace Transform** and its domain.  
(Please use WolframAlpha to check your answer. Type 'Laplace Transform of ...')

a.  $f(t) = 1$  (Optional: watch <https://www.khanacademy.org/math/differential-equations/laplace-transform/laplace-transform-tutorial/v/laplace-transform-1>)

Domain of F =

b.  $f(t) = e^t$  (Optional: watch <https://www.khanacademy.org/math/differential-equations/laplace-transform/laplace-transform-tutorial/v/laplace-transform-2>)

Domain of F =

c.  $f(t) = t$

Domain of F =