7.2 Trigonometric Integrals

Trigonometric Identities and Formulas.

1.) $\sin^2\theta + \cos^2\theta = 1$

Why?

3.) $\sin(2\theta) = 2\sin\theta\cos\theta$

Why?

5.) $\cos^2\theta = \dfrac{1+\cos(2\theta)}{2}$

Why?

2.) $\tan^2\theta + 1 = \sec^2\theta$

Why?

4.) $\cos(2\theta) = \cos^2\theta - \sin^2\theta$

Why?

https://www.khanacademy.org/math/precalculus/trig-equations-and-identities-precalc/intro-to-trig-angle-addition-identities-precalc/v/proof-angle-addition-cosine

or differentiate both sides of identity (3).

6.) $\sin^2\theta = \dfrac{1-\cos(2\theta)}{2}$

Why? **HW**

Example: Evaluate $\int_0^{\pi} \sin^3(5x)\,dx$.

Thinking about the problem:

To integrate a power like $\sin^3(5x)$, let's write $\sin^3\theta$ in terms of lower powers. By the first trigonometric identity above, we can write $\sin^2\theta = 1 - \cos^2\theta$, so

$$\sin^3\theta = \sin^2\theta \sin\theta = (1 - \cos^2\theta)\sin\theta.$$

Therefore (using $\theta = 5x$)

$$\int_0^{\pi} \sin^3(5x)\,dx = \int_0^{\pi} (1 - \cos^2(5x))\sin(5x)\,dx.$$

Doing the problem:

After rewriting of the function being integrated, let's use the substitution $u = \cos(5x)$, so $du = -5\sin(5x)\,dx$:

$$\int (1-\cos^2(5x))\sin(5x)\,dx = \int (1-u^2)\frac{-du}{5} = -\frac{1}{5}\int (1-u^2)\,du.$$

Let's turn x-bounds into u-bounds in the definite integral:

$$x = 0 \Longrightarrow u = \cos(5\cdot 0) = \cos 0 = 1, \quad x = \pi \Longrightarrow u = \cos(5\pi) = -1.$$

Therefore

$$\begin{aligned}
\int_0^\pi \sin^3(5x)\,dx &= \int_{x=0}^{x=\pi} (1-\cos^2(5x))\sin(5x)\,dx \\
&= -\frac{1}{5}\int_{u=1}^{u=-1} (1-u^2)\,du \quad \text{(Note the order of integration)} \\
&= \frac{1}{5}\int_{-1}^{1} (1-u^2)\,du \quad \text{(Sign change in the order of integration)} \\
&= \frac{1}{5}\left(u-\frac{u^3}{3}\right)\Bigg|_{-1}^{1} \\
&= \frac{1}{5}\left(\left(1-\frac{1}{3}\right)-\left(-1-\frac{-1}{3}\right)\right) \\
&= \frac{1}{5}\left(1-\frac{1}{3}+1-\frac{1}{3}\right) \\
&= \frac{1}{5}\left(2-\frac{2}{3}\right) \\
&= \frac{4}{15}.
\end{aligned}$$

Solutions should show all of your work, not just a single final answer.

1. Show the formulas $\sin(2x) = 2\sin x \cos x$ and $\cos(2x) = \cos^2 x - \sin^2 x$ each imply the other one using differentiation: differentiate each identity and simplify to turn the first formula into the second and the second formula into the first.

   - Part A: Show that $\sin(2x) = 2\sin x \cos x$ implies $\cos(2x) = \cos^2 x - \sin^2 x$.

   - Part B: Show that $\cos(2x) = \cos^2 x - \sin^2 x$ implies $\sin(2x) = 2\sin x \cos x$.

2. Identify the trigonometric identities to simplify the following integrands, and carry out the integration. Remember to include $+C$ in the final answer.

   (a) $\displaystyle\int \sin^2 x \, dx$

   (b) $\displaystyle\int \cos^2 x \, dx$

3. Evaluate $\int \sin^2 x \cos^2 x \, dx$ by using a trigonometric identity involving $\sin x \cos x$ to simplify the integrand.

4. Evaluate the definite integral $\int_0^{\pi} \sin^3 x \, dx$.

5. Evaluate $\int \cos x \sin^2 x\, dx$. (There may be more than one technique that works.)

6. T/F (with justification): The value of $\int_{-\pi}^{\pi} \sin^9 x\, dx$ is 0.