HW due Week 6 Day 3

Name : ______________________________

Each of the following integrals can be solved using trig substitution. Do at least FIVE of them. You should sketch a triangle for each problem. Full solutions are available on the course website.

1. Evaluate $\int \frac{1}{x\sqrt{4-x^2}}\,dx$.

[Solution] $= -\frac{1}{2}\ln\left|\frac{2+\sqrt{4-x^2}}{x}\right| + C$

2. Evaluate $\int \frac{1}{\sqrt{x^2+16}}\,dx$.

[Solution] $= \ln\left|\frac{\sqrt{x^2+16}+x}{4}\right| + C$

3. Evaluate $\int_{\sqrt{2}}^{2} \frac{1}{x^3\sqrt{x^2-1}}\,dx$.

[Solution] $= \left(\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right)_{\frac{\pi}{4}}^{\frac{\pi}{3}}$

$= \frac{\pi}{24} + \frac{\sqrt{3}}{8} - \frac{1}{4}$

4. Evaluate $\int_{0}^{\frac{\pi}{2}} \frac{\cos x}{\sqrt{1+\sin^2 x}}\,dx$.

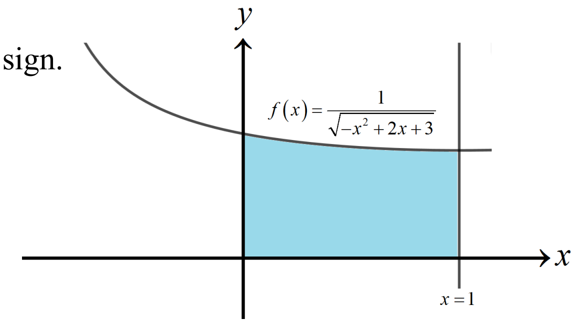
[Solution] Tip: u-substitution, then trig substitution with u=tan theta.

Answer $= \left(\ln\left|\sec\theta + \tan\theta\right|\right)_{0}^{\frac{\pi}{4}}$

$= \ln\left(\sqrt{2}+1\right)$

5. Let R be the region bounded by the function $f(x)=\dfrac{1}{\sqrt{-x^2+2x+3}}$ and x-axis on the interval $[0,1]$. Find the area of the region R.

Tip: First complete the square under the root sign.
See Sec 7.3 Example 7, pg 490.
You can directly do inverse trig sub
or first do u-substitution with u = (x-1)/2



[Solution 1]

$$\text{Area} = \int_0^1 \frac{1}{\sqrt{4-(x-1)^2}}\,dx$$
$$= \int_{-\frac{\pi}{6}}^{0} 1\,d\theta$$
$$= \frac{\pi}{6}$$

[Solution 2]

$$\text{Area} = \int_0^1 \frac{1}{2\sqrt{1-\left(\frac{x-1}{2}\right)^2}}\,dx$$
$$= \left(\sin^{-1} u\right)_{-\frac{1}{2}}^{0}$$
$$= \frac{\pi}{6}$$

6. Evaluate $\displaystyle\int \frac{1}{\sqrt{1+16x^2}}\,dx$. **[Solution]** $=\dfrac{1}{4}\ln\left|\sqrt{1+16x^2}+4x\right|+C$

7. Evaluate $\displaystyle\int_0^{\sqrt{2}} \frac{x^2}{\sqrt{4-x^2}}\,dx$. **[Solution]** $=\left(2\theta-\sin 2\theta\right)_0^{\frac{\pi}{4}}=\dfrac{\pi}{2}-1$

8. Evaluate $\displaystyle\int \frac{1}{x^2\sqrt{9x^2-1}}\,dx$. **[Solution]** $=3\sin\theta+C=\dfrac{\sqrt{9x^2-1}}{x}+C$

9. Evaluate $\displaystyle\int \sqrt{5+4x-x^2}\,dx$. **[Solution]** $=\dfrac{9}{2}\theta+\dfrac{9}{2}\sin\theta\cos\theta+C$

$$=\frac{9}{2}\sin^{-1}\left(\frac{x-2}{3}\right)+\frac{9}{2}\cdot\frac{x-2}{3}\cdot\frac{\sqrt{9-(x-2)^2}}{3}+C=\frac{9}{2}\sin^{-1}\left(\frac{x-2}{3}\right)+\frac{x-2}{2}\sqrt{9-(x-2)^2}+C$$