Worksheet

Name : ________________________________

The final answers are given for each problem. Your job is to provide the explanations. Use your own paper.

1. Find the interval and radius of convergence for the power series.

Choose at least one series from (a)-(f) (choose the most challenging).

a. $\sum_{n=0}^{\infty} \frac{(-2)^n (x+3)^n}{3^{n+1}}$

Answer: Radius of convergence is R = 3/2 and interval of convergence is I = (-9/2, -3/2) both open.

b. $\sum_{n=1}^{\infty} (-1)^n nx^n$

Answer: Radius of convergence is R = 1 and interval of convergence is I = (-1, 1) both open.

c. $\sum_{n=1}^{\infty} \frac{(-1)^n (x-3)^n}{\sqrt{n}}$

Answer: Radius of convergence is R = 1 and interval of convergence is I = (open) (2, 4] (closed)

d. $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{7^n (n+1)}$

Answer: Radius of convergence is R = 7 and interval of convergence is I = (open) (-7, 7] (closed)

e. $\sum_{n=1}^{\infty} (-1)^n \frac{n^2 x^n}{2^n}$

Answer: Radius of convergence is R = 2 and interval of convergence is I = (-2, 2) both open

f. $\sum_{n=1}^{\infty} \frac{10^n x^n}{n^3}$

Answer: Radius of convergence is R = 1/10 and interval of convergence is I = [-1/10, 1/10] both closed

Choose at least one series from (g)-(l). Choose the most challenging series.

g. $\sum_{n=0}^{\infty} \frac{n!(x-2)^n}{3^n}$

Answer: Radius of convergence is R = 0 and interval of convergence is I = {2}

h. $\sum_{n=1}^{\infty} (-1)^n \frac{x^n}{4^n \ln n}$

Answer: Radius of convergence is R = 4 and interval of convergence is I = open (-4, 4] closed

i. $\sum_{n=1}^{\infty} \frac{n^{20} x^n}{(2n+1)!}$

Answer: interval of convergence is all real numbers

j. $\sum_{n=0}^{\infty} \frac{(x-2)^n}{n^2+1}$

Answer: Radius of convergence is R = 1 and interval of convergence is I = closed [1, 3] closed

k. $\sum_{n=1}^{\infty} n!(2x-1)^n$

Answer: Radius of convergence is R = 0 and interval of convergence is I = {1/2}

l. $\sum_{n=2}^{\infty} \frac{x^{2n}}{n(\ln n)^2}$

Answer: Radius of convergence is R = 1 and the interval of convergence is I = [-1,1] both closed

Choose at least one series from no. 2-3. Choose the most challenging series.

2. Find the radius of the convergence and the interval of convergence for the series $\sum_{n=2}^{\infty} \frac{1}{n \ln n}\left(\frac{x}{2}-1\right)^n$.

Answer: radius of convergence is R = 2 and the interval of convergence is (closed) [0,4) (open).

3. The following functions can be represented as power series. Find the interval of convergence for the power series**.**
   a. $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$

Answer: the interval of convergence is (-1,1)

   b. $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

Answer: all real numbers

   c. $\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$

Answer: all real numbers

   d. $\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$

Answer: all real numbers

Find the interval of convergence for *both* series.

   e. $\arctan x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$

Answer: the interval of convergence is [-1,1] both closed.

   f. $\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$

Answer: the interval of convergence is open (-1, 1] closed.