Names (please print): KEY **Math 1152Q: Fall '18**

Sec 11.4 in-class "Homework" KEY, Monday, Feb 5

1. Determine whether each series converges or diverges.

$$a.)\quad \sum_{n=1}^{\infty} \frac{\ln\ n}{n} \qquad\qquad b.)\quad \sum_{n=4}^{\infty} \frac{1}{2^n - 9}$$ (See also book answers: Sec 11.4 Examples 2 and 3).

Try Divergence Test: Both $\frac{ln\ n}{n}$ and $\frac{1}{2^n-9}$ converge to 0. So the Divergence Test is inconclusive.

Try Limit Comparison Test (LCT):

For $a_n = \frac{\ln\ n}{n}$:

Try $b_n = \frac{1}{n}$.

$$\begin{aligned} \lim_{n\to\infty} \frac{a_n}{b_n} &= \lim_{n\to\infty} \frac{\ln n}{n} \frac{n}{1} \\ &= \lim_{n\to\infty} \ln n = \infty. \end{aligned}$$

Since $\sum b_n$ diverges by the harmonic series test (p-series test), by the Limit Comparison Test, we conclude that $\sum a_n$ also diverges. (See also book's solution using Comparison Test. Sec 11.4 Example 2 pg 728-729.)

For $a_n = \frac{1}{2^n-9}$:

Try $b_n = \frac{1}{2^n}$

$$\begin{aligned} \lim_{n\to\infty} \frac{a_n}{b_n} &= \lim_{n\to\infty} \frac{1}{2^n - 9} \frac{2^n}{1} \\ &= \lim_{n\to\infty} \frac{2^n}{2^n - 9} \\ &= \lim_{n\to\infty} \frac{1}{1 - \frac{9}{2^n}} \\ &= \frac{1}{1 - \lim_{n\to\infty} \frac{9}{2^n}} = 1, \qquad \text{which is a positive number.} \end{aligned}$$

We know that $\sum b_n$ converges by the geometric series test (or you can show this using the ratio test or root test). So, by the Limit Comparison test, we conclude that $\sum a_n$ also converges. (Warning: if $\lim\limits_{n\to\infty} \frac{a_n}{b_n}$ had been ∞, we cannot conclude anything).

(For now, you can ignore this future topic) Try Ratio test:

For $a_n = \frac{\ln\ n}{n}$: you know that ratio tests will be *inconclusive* for terms involving only constants, log, and polynomials. See 1st page of "growth rates" notes: `https://egunawan.github.io/spring18/notes/notes11_6part3key.pdf`

For $a_n = \frac{1}{2^n-9}$:

$$\begin{aligned} \lim_{n\to\infty} \frac{a_{n+1}}{a_n} &= \lim_{n\to\infty} \frac{2^n - 9}{2^{n+1} - 9} \\ &\overset{L}{=} \lim_{n\to\infty} \frac{\ln 2\ 2^n}{\ln 2\ 2^{n+1}} \\ &= \frac{1}{2} < 1. \end{aligned}$$

By the ratio test, $\sum_{n=4}^{\infty} \frac{1}{2^n-9}$ converges.

2. Determine whether each series is convergent or divergent.

$$i.)\quad \sum_{n=1}^{\infty} \frac{2n^2+3n}{\sqrt{5+n^5}} \qquad ii.)\quad \sum_{n=1}^{\infty} \frac{5}{2n^2+4n+3} \qquad \text{(from Sec 11.4 Examples 1 and 4).}$$

Try Divergence Test: Both $\frac{2n^2+3n}{\sqrt{5+n^5}}$ and $\frac{5}{2n^2+4n+3}$ converge to 0. The Divergence Test is inconclusive.

Try Limit Comparison Test (LCT):

For $a_n = \frac{2n^2+3n}{\sqrt{5+n^5}}$**:**

Try $b_n = \frac{1}{\sqrt{n}}$.

See textbook Sec 11.4 Example 4 page 730 for solution.

For $a_n = \frac{5}{2n^2+4n+3}$**:**

Try $b_n = \frac{1}{n^2}$.

$$\begin{aligned}
\lim_{n\to\infty} \frac{a_n}{b_n} &= \lim_{n\to\infty} \frac{5}{2n^2+4n+3}\frac{n^2}{1} \\
&= \lim_{n\to\infty} \frac{5n^2}{2n^2+4n+3} \\
&= \lim_{n\to\infty} \frac{5}{2+\frac{4n}{n^2}+\frac{3}{n^2}} \\
&= \frac{5}{2+\lim_{n\to\infty}\frac{4}{n}+\lim_{n\to\infty}\frac{3}{n^2}} \\
&= \frac{5}{2+0+0} \\
&= \frac{5}{2}, \qquad \text{which is a positive number.}
\end{aligned}$$

Since $\sum b_n$ converges by the p-series test, by the Limit Comparison Test, we conclude that $\sum a_n$ also converges.

(See also textbook Sec 11.4 Example 1 page 728 for solution using the comparison test. I think my LCT solution will takes less time to do in case the term of your series has minus signs in the denominator.)

(For now, you can ignore this future topic) Try Ratio Test (for both series): You will find that the Ratio Test is inconclusive. Recall that ratio tests will be inconclusive for series that look like p-series, so you don't even need to try.