

Name : _____

So far we have learned three tests for testing convergence/divergence of a **SERIES**:

- **Geometric Series Test:** a geometric series is convergent if and only if the ratio is between -1 and 1.
- **Divergence Test:** if the sequence does not converge to 0, the series diverges.
- **Harmonic Series Test:** the harmonic series diverges.

Please verify your answer with WolframAlpha:

<https://www.wolframalpha.com/examples/math/calculus/sequences/>

1. (Do at least two - pick the most challenging-looking series)

Determine whether the **geometric series** is convergent or divergent. If it is convergent, find its sum.

a. $3 - 4 + \frac{16}{3} - \frac{64}{9} + \dots$

b. $2 + 0.5 + 0.125 + 0.03125 + \dots$

c. $\sum_{k=1}^{\infty} \frac{10^k}{(-9)^{k-1}}$

d. $\sum_{k=1}^{\infty} \frac{(-3)^{k+1}}{4^k}$

e. $\sum_{k=0}^{\infty} (-\pi)^k e^{-k}$

f. $\sum_{k=2}^{\infty} \frac{2^{3k+1}}{3^{2k-1}}$

2. Consider the infinite series $\sum_{k=1}^{\infty} \ln\left(\frac{k+1}{k}\right)$.
- Determine whether the sequence $\{a_n\}$ is convergent or divergent.
 - Determine whether the series $\sum_{n=1}^{\infty} a_n$ is convergent or divergent (if possible).

Determine whether we can apply one of the three tests we've learned.

- Can we apply the Geometric Series test?
- Is the Divergence Test conclusive?
- Can we apply the Harmonic Series Test?

3. Consider the infinite series $\sum_{k=1}^{\infty} \frac{1}{k(k+1)}$.
- Determine whether the sequence $\{a_n\}$ is convergent or divergent.
 - Determine whether the series $\sum_{n=1}^{\infty} a_n$ is convergent or divergent (if possible).

Determine whether we can apply one of the three tests we've learned.

- Can we apply the Geometric Series Test?
- Is the Divergence Test conclusive?
- Can we apply the Harmonic Series Test?

4. Let $a_n = \frac{n}{2n+1}$
- Determine whether the sequence $\{a_n\}$ is convergent or divergent.
 - Determine whether the series $\sum_{n=1}^{\infty} a_n$ is convergent or divergent (if possible).

Determine whether we can apply one of the three tests we've learned.

- Can we apply the Geometric Series Test?
- Is the Divergence Test conclusive?
- Can we apply the Harmonic Series Test?

5. (Do at least one) Attempt to determine whether the series $\sum_{n=1}^{\infty} a_n$ is convergent or divergent using some of the three tests we've learned (possibly none of them works). If it is convergent (geometric series), find its sum.

a. $\frac{1}{3} + \frac{1}{6} + \frac{1}{9} + \frac{1}{12} + \frac{1}{15} + \dots$

b. $\frac{1}{3} + \frac{2}{9} + \frac{1}{27} + \frac{2}{81} + \frac{1}{243} + \frac{2}{729} + \dots$

c. $\sum_{n=1}^{\infty} \frac{n(n+2)}{(n+3)^2}$

6. (Do at least one) Attempt to determine whether the series $\sum_{n=1}^{\infty} a_n$ is convergent or divergent using some of the three tests we've learned (possibly none of them works). If it is convergent (geometric series), find its sum. You can use Desmos/ WolframAlpha to plot your sequence.

d. $\sum_{n=1}^{\infty} \frac{2^n + 3^n}{5^n}$

e. $\sum_{n=1}^{\infty} \sqrt[n]{2}$

f. $\sum_{n=1}^{\infty} \ln\left(\frac{n+1}{2n+1}\right)$

7. (Do at least one) Attempt to determine whether the series $\sum_{n=1}^{\infty} a_n$ is convergent or divergent using some of three tests we've learned (possibly none of them works). If it is convergent (geometric series), find its sum. You can use Desmos/ WolframAlpha to plot your sequence.

g.
$$\sum_{n=1}^{\infty} \frac{1}{1 + \left(\frac{2}{3}\right)^n}$$

h.
$$\sum_{n=1}^{\infty} (\cos 1)^n$$

i.
$$\sum_{n=1}^{\infty} \arctan n$$

8. (Do at least one) Attempt to determine whether the series $\sum_{n=1}^{\infty} a_n$ is convergent or divergent using the geometric series test or divergence test (possibly none of them works). If it is convergent (geometric series), find its sum.

j. $\sum_{n=1}^{\infty} \left(\frac{3}{5^n} + \frac{2}{n} \right)$

k. $\sum_{n=1}^{\infty} \frac{2^n}{n^2}$

l. $\sum_{n=1}^{\infty} n \sin\left(\frac{1}{n}\right)$

m. $\sum_{n=1}^{\infty} \left[\frac{5}{n(n+1)} - (-1)^n \frac{3}{2^n} \right]$