Comparison Tests

Solutions should show all of your work, not just a single final answer.

1. State the comparison test and the limit comparison test.

2. Determine if the following series converge or diverge using a comparison test with some p-series. Carry out the test with clear explanations.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+n}$. (This could also be settled by writing $\dfrac{1}{n^2+n} = \dfrac{1}{n(n+1)}$ as $\dfrac{1}{n} - \dfrac{1}{n+1}$ and using a telescoping series.)

   (b) $\displaystyle\sum_{n=2}^{\infty} \frac{\ln n}{\sqrt{n}}$.

3. Determine whether the following infinite series converge or diverge using the limit comparison test. Carry out the test with clear explanations.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2+1}$. (This was already done with the Integral Test in Worksheet 11.3.)

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n-n}$.

4. T/F (with justification)

   The divergence of p-series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^p}$ for $0 < p < 1$ follows from divergence of the harmonic series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ by the comparison test.

5. T/F (with justification)

   The convergence of p-series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^p}$ for $p > 1$ follows from divergence of the harmonic series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ by the comparison test.