Sections 11.1-11.2, 11.4-11.6

1. For the following questions, circle TRUE or FALSE, and give a justification. True statements should be argued for using facts, theorems or definitions from class.

   (a) If $\lim_{n\to\infty} a_n = 0$ then the series $\sum a_n$ converges. **T** **F**

   Justification:

   (b) If $a_n > 0$, $b_n > 0$ for all n, $\sum b_n$ diverges, and $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$, then $\sum a_n$ diverges. **T** **F**

   Justification:

   (c) If $a_n > 0$ for all n & $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 0$, then $\sum a_n$ is convergent by the ratio test **T** **F**

   Justification:

   (d) If a_n and b_n are both positive for all n and $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$, then $\sum a_n$ is convergent by the limit comparison test **T** **F**

   Justification:

   (e) The harmonic series $\sum 1/n$ is convergent by the p-series test **T** **F**

   Justification:

   (f) We can use the ratio test *alone* to show the geometric series $\sum \frac{2^n}{3^n}$ converges **T** **F**

   Justification:

   (g) We can use the p-series test *alone* to show the series $\sum 2^n/3^n$ converges **T** **F**

   Justification:

   (h) We can apply the monotonic sequence theorem to show that the geometric **sequence** $\{2^n/3^n\}_{n=1}^{\infty}$ is convergent **T** **F**

   Justification:

   (i) We can apply the monotonic sequence theorem to show that the harmonic sequence $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$ is convergent **T** **F**

   Justification:

   (j) We can apply the squeeze theorem to show that the alternating harmonic sequence $\left\{\frac{(-1)^n}{n}\right\}$ is convergent **T** **F**

   Justification:

   (k) It is impossible for a subset of a line to have infinitely many points and have length zero. **T** **F**

   Justification:

   (l) The divergence of the p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ for $0 < p < 1$ follows from divergence of the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ and the comparison test. **T** **F**

   Justification:

   (m) The convergence of the p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ for $p > 1$ follows from divergence of the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ and the comparison test. **T** **F**

   Justification:

   (n) Convergence of $\sum \frac{1}{n^p}$ for $p > 1$ can be shown with the ratio test. **T** **F**

   Justification:

   (o) Divergence of a p-series for $p < 1$ can be shown with the ratio test. **T** **F**

   Justification:

2. (a) State the contrapositive of the factual statement: "If the sequence $\{a_n\}$ is unbounded, then it is divergent".

   (b) Is the contrapositive statement you wrote as your answer to part (a) true or false? **Justification (explain or give a counterexample):**

   (c) The converse of part (a) is the following: "If the sequence $\{a_n\}$ is divergent, then $\{a_n\}$ is unbounded". Is this true or false? **Justification (explain or give a counterexample):**

3. Answer the following on the line provided.

(a) What is the 100th term of the sequence $\{2, 5, 8, 11, \ldots\}$?
(The terms 2 and 5 are the first and second term, respectively) ____________

(b) Find a formula for the general term a_n of the sequence $\left\{1, -\frac{2}{5}, \frac{3}{25}, -\frac{4}{125}, \frac{5}{625} \ldots\right\}$. Make sure to specify your starting value of n.

(c) Write the geometric series $4 + 2 + 1 + \frac{1}{2} \cdots$ in standard form.
(summation notation $\sum$) ____________

(d) Find the 4th term a_4 in the recursive sequence $a_{n+1} = 2a_n + a_{n-1}$ when $a_1 = 1$ and $a_2 = 1$. ____________

(e) Find the 7th term a_7 in the recursive sequence $a_{n+1} = a_n + a_{n-1}$ when $a_1 = 2$ and $a_2 = 3$. ____________

(f) We can use geometric series to compute

$$\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \ldots.$$

What fraction is this equal to?

(g) We can use geometric series to compute $0.9999\ldots.$ What fraction is this equal to?

(h) One of the two decimal expansions for a number is $2.449999\ldots.$ What's the other?

(i) Use geometric series to compute the fraction for $1.833333\ldots.$
Perform a sanity check against your answer.

(j) Use geometric series to compute the fraction for $1.08333333333333333\ldots$
Perform a sanity check against your answer.

(k) Does the series $\sum_{n=1}^{\infty} -\ln\left(\frac{n}{2n+7}\right)$ converge or diverge? ____________

(l) Find the sum of the series $\sum_{k=0}^{\infty} 5\left(\frac{2}{3}\right)^k$. ____________

(m) Find the sum of the series $\sum_{n=2}^{\infty} 5\left(\frac{(-6)^{n-1}}{7^n}\right)$. ____________
Perform a sanity check against your answer.

(n) Find the sum of the series ____________

$$-5 + 3 - \frac{9}{5} + \frac{27}{25} - \frac{81}{125} + \ldots.$$

Perform a sanity check against your answer.

(o) Write an expression for the nth term in the sequence ____________
$\left\{\frac{1}{2}, \frac{1}{6}, \frac{1}{24}, \frac{1}{120}, \ldots\right\}$. (The terms $\frac{1}{2}$ and $\frac{1}{6}$ are the first and second terms in the sequence)

(p) Write an equivalent series with index summation beginning at $n = 0$. ____________

$$\sum_{n=2}^{\infty} \frac{2^n}{(n-2)!}$$

Perform a sanity check against your answer.

(q) Write an equivalent series with index summation beginning at $n = 1$. ____________

$$\sum_{n=2}^{\infty} \frac{2^n}{(n-2)!}$$

Perform a sanity check against your answer.

(r) For what values of k does the series $\sum \frac{5}{n^k}$ converge? Please explain. ____________

(s) Find the values of A so that the series $\sum_{n=1}^{\infty} \frac{(A)^{n-1}}{3^{n-1}}$ is convergent. Please explain.

(t) Find the values of B so that the series $\sum_{n=1}^{\infty} \frac{(B-3)^{n-1}}{3^{n-1}}$ is convergent. Please explain.

(u) Find the values of C so that the series $\sum_{n=5}^{\infty} \frac{(C-2)^{n}}{3^{n+1}}$ is convergent. Please explain.

4. The following questions ask you to determine the converge/divergence of a series. To receive credit, give detailed explanation (for example, follow my answer key - to be posted).

   (a) Determine whether the series $\sum_{k=1}^{\infty} \frac{k^k}{7^k (k)!}$ is convergent.

   (b) Determine whether the series $\sum_{k=1}^{\infty} \frac{k^k}{2^k (k)!}$ is convergent.

   (c) Determine whether the series $\sum_{n=3}^{\infty} \frac{6}{n\sqrt{n^2-8}}$ converges or diverges.

   (d) Determine whether the series $\sum_{n=1}^{\infty} \frac{\sqrt{n^3+1}}{3n^3-4n+2}$ converges or diverges. Make sure to state which test/s you use and provide justifying computations and arguments.

   (e) Determine whether the series $\sum_{n=1}^{\infty} \frac{n!}{7^n (n+8)!}$ converges or diverges. Make sure to state which test/s you use and provide justifying computations and arguments.

   (f) Determine whether the series $\sum_{n=1}^{\infty} \frac{(n+8)!}{7^n n!}$ converges or diverges. Make sure to state which test/s you use and provide justifying computations and arguments. (This looks similar to above, but this isn't a typo).

   (g) Determine whether the series $\sum_{n=1}^{\infty} \ln\left(\frac{3n}{n+1}\right)$ converges or diverges. Make sure to state which test/s you use and provide justifying computations and arguments.

   (h) Determine whether the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{5n+3}$ converges or diverges. Make sure to state which test/s you use and provide justifying computations and arguments.

   (i) Determine the convergence of $\sum_{k=1}^{\infty} k \cos\left(\frac{\pi k+1}{2k}\right)$.

   (j) Determine the convergence of $\sum_{k=1}^{\infty} \left(\frac{2k}{5k+5} + \frac{1}{(4)^k}\right)$

   (k) Determine the convergence of $\sum_{k=1}^{\infty} \frac{2^{4k+1}}{5^{2k-1}}$. If this series is convergent, compute its sum.

   (l) Determine the convergence of $\sum_{k=2}^{\infty} \frac{8^{3k+1}}{9^{2k-1}}$. If this series is convergent, compute its sum.

5. Review pg 4-5 of `https://egunawan.github.io/spring18/notes/notes4_4lhopitals_rule.pdf`

   (a) Compute $\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^{3n}$.

   (b) Compute $\lim_{n\to\infty} \left(1+\frac{1}{2n}\right)^{n}$.

   (c) Compute $\lim_{n\to\infty} \left(1+\frac{2}{n}\right)^{5n}$.

(d) Compute $\lim_{n\to\infty} n^2 e^{-n}$.

(e) Compute $\lim_{n\to\infty} \frac{\ln n}{n}$.

(f) Compute $\lim_{n\to\infty} \frac{n \sin n}{n^2+1}$.

6. Consider the series $\sum a_n = \sum_{n=2}^{\infty} \frac{\cos(n\pi)}{n-1}$.

(a) What are the first three terms in the series? ________________

(b) Is the series convergent? You must justify. ________________

(c) Is the series $\sum a_n = \sum_{n=2}^{\infty} \frac{|\cos(n\pi)|}{n-1}$ convergent? You must justify. ________________

7. (a) Define an alternating series.

(b) State the alternating series test.

(c) Determine whether the series

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots = \sum_{k=1}^{\infty} \frac{(-1)^{n-1}}{n}$$

is convergent or divergent.

(d) Determine whether the series $\sum_{k=1}^{\infty} \frac{(-1)^n \, 3n}{4n-1}$ is convergent or divergent.

(e) Determine whether the series $\sum_{k=1}^{\infty} (-1)^{n+1} \frac{n^2}{n^3+1}$ converges or diverges.

8. (a) Evaluate $\lim_{n\to\infty} e^{-n}\sqrt{n}$.

(b) Determine whether $\sum_{n=0}^{\infty} e^{-n}\sqrt{n}$ converges or diverges.

(c) Evaluate $\lim_{n\to\infty} \frac{(\ln(n))^2}{n^2}$.

(d) Determine whether $\sum_{n=1}^{\infty} \frac{(\ln(n))^2}{n^2}$ converges or diverges.

(e) Suppose $\sum_{n=1}^{\infty} a_n$ is a series with the property that

$$a_1 + a_2 + \cdots + a_n = 2 - 3(0.8)^n.$$

State whether $\sum_{n=1}^{\infty} a_n$ converges or diverges. If it converges, find its sum.

9. Determine whether each series converges or diverges.

a.) $\sum_{n=1}^{\infty} \frac{\ln\ n}{n}$ *b.)* $\sum_{n=4}^{\infty} \frac{1}{2^n - 9}$ *i.)* $\sum_{n=1}^{\infty} \frac{2n^2+3n}{\sqrt{5+n^5}}$ *ii.)* $\sum_{n=1}^{\infty} \frac{5}{2n^2+4n+3}$