Useful trig facts.

$$\sin^2\theta + \cos^2\theta = 1, \quad \tan^2\theta + 1 = \sec^2\theta, \quad \cos^2\theta = \frac{1}{2}(1+\cos 2\theta), \quad \sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta, \quad \sin 2\theta = 2\sin\theta\cos\theta$$

$$\sin\frac{\pi}{6} = \frac{1}{2}, \quad \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}, \quad \sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}, \quad \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}, \quad \cos\frac{\pi}{3} = \frac{1}{2},$$

Some derivatives and antiderivatives.

$\frac{d}{dx}\sin(x) = \cos(x)$	$\frac{d}{dx}\cos(x) = -\sin(x)$	$\frac{d}{dx}\tan(x) = (\sec(x))^2$
$\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$	$\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$	$\frac{d}{dx}\cot(x) = -(\csc(x))^2$
$\frac{d}{dx}\arcsin(x) = \frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}\arccos(x) = \frac{-1}{\sqrt{1-x^2}}$	$\frac{d}{dx}\arctan(x) = \frac{1}{1+x^2}$
$\frac{d}{dx}b^x = \ln(b)b^x$	$\int \sec x \text{ dx} = \ln\lvert\sec x + \tan x\rvert + C.$	$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C$ if $a \neq 0$.

Fundamental Theorem of Calculus, part I.

Part 1: If f is continuous on $[a,b]$, then function g defined as

$$g(x) = \int_a^x f(t)\,dt, \quad a \le x \le b$$

satisfies $g'(x) = f(x)$.

Fundamental Theorem of Calculus, part II.

If f is continuous on $[a,b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

where F is any anti-derivative of f (ie. F is any function such that $F' = f$).

Integration by parts fomula.

$$\int u\,dv = uv - \int v\,du$$

Restricted domains for trig functions

$\sin\theta$ for $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$	$\cos\theta$ for $0 \le \theta \le \pi$	$\tan\theta$ for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$
$\csc\theta$ for θ in $[-\pi/2, 0) \cup (0, \pi/2]$	$\sec\theta$ for θ in $[0, \pi/2) \cup (\pi/2, \pi]$	$\cot\theta$ for $0 < \theta < \pi$

Partial Fraction Decomposition.

$$\frac{1}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b} \text{ if } a \neq b, \text{ and } \qquad \frac{1}{x(x^2+a)} = \frac{A}{x} + \frac{Bx+C}{x^2+a} \text{ if } a \neq 0$$

The Integral Test.

If $a_n = f(n)$ where $f(x)$ is a continuous, positive, decreasing function for $x \geq 1$, then $\sum_{n=1}^{\infty} a_n$ is convergent if and only if the improper integral $\int_1^{\infty} f(x)\, dx$ is convergent.

Remainder Estimate for the Integral Test.

If $a_n = f(n)$, where $f(x)$ is a continuous, positive, decreasing function for $x \geq 1$ as above, suppose $S = \sum_{n=1}^{\infty} a_n$ converges. For $N \geq 1$, let $s_N = \sum_{n=1}^{N} a_n$ and $R_N = S - S_N$, so R_N is the Nth remainder term. Then

$$\int_{N+1}^{\infty} f(x)\, dx \leq R_N \leq \int_{N}^{\infty} f(x)\, dx.$$

Alternating series test.

A series of the form

$$\sum_{n=1}^{\infty} (-1)^{n+1} b_n \quad \text{or} \quad \sum_{n=1}^{\infty} (-1)^n b_n$$

where $b_n \geq 0$ for all n is called an alternating series. If

1. $b_{n+1} \leq b_n$ for all n large enough (ie. $\{b_n\}$ is an eventually decreasing sequence)
2. $\lim_{n\to\infty} b_n = 0$

then the series **converges**.

Alternating Series Estimation Theorem

If $S := \sum_{k=r}^{\infty} (-1)^k b_k$, where $b_k > 0$, is the sum of an alternating series that satisfies

$$\text{(i) } b_{k+1} \leq b_k \qquad \text{and} \qquad \text{(ii) } \lim_{k\to\infty} b_k = 0,$$

then $|R_N| = |S - S_N| \leq b_{N+1}$, where $S_N := \sum_{k=r}^{N} (-1)^k b_k$.