

MATH 1152Q Exam 1 Summary

[11.1-11.2, 11.4-11.6] Infinite Sequences and Series

【11.1】 Sequences

1. Determine whether the **sequence** $\{a_n\}$ is convergent to a number.

a. $a_n = \frac{3^{n+2}}{5^n}$

b. $a_n = \sqrt[2n]{e^{n+2}}$

c. $a_n = \frac{(\ln n)^2}{n}$

d. $a_n = \frac{\cos^2 n}{2^n}$

【11.2】 Series

1. Determine if the series is convergent or divergent. If it is convergent, find its sum.

a. $\sum_{n=1}^{\infty} \left(\frac{e}{\pi}\right)^n$. You don't need to simplify.

b. $\sum_{n=1}^{\infty} \ln\left(\frac{n+1}{n}\right)$. Hint: write as a telescoping sum.

c. $\sum_{n=1}^{\infty} \left[\left(\frac{e}{\pi}\right)^n + \ln\left(\frac{n+1}{n}\right) \right]$

2. Express 0.10181818181818 ... as a ratio of two integers. You don't need to simplify.

【11.2】 The Divergence Test

1. Let $a_n = \frac{n^2}{2n+1} - \frac{n^2}{2n-1}$.

a. Determine whether the sequence $\{a_n\}$ is convergent or divergent. Hint: simplify.

b. Determine whether the series $\sum_{n=1}^{\infty} a_n$ is convergent or divergent.

【11.6】 The Ratio Test

1. Use the **Ratio Test** to determine whether the series $\sum_{n=1}^{\infty} \frac{n3^n}{(2n+1)!}$ is convergent.

【11.4】 The 2 Comparison Tests

1. Use the **Limit Comparison Test (or the comparison test)** to determine whether the series $\sum_{n=1}^{\infty} \frac{\cos^2 n + 5n}{8^n}$ is convergent or divergent.
2. Use the **Limit Comparison Test (or the comparison test)** to determine whether the series $\sum_{n=1}^{\infty} \frac{\sqrt[3]{n}}{\sqrt{n^3 + 4n + 3}}$ is convergent or divergent.
3. Consider the series $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$.
 - a. Use the **Divergence Test** to determine whether the series $\sum_{n=1}^{\infty} n \sin\left(\frac{1}{n}\right)$ is convergent.
 - b. Use the **Limit Comparison Test** to determine whether the series $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$ is convergent or divergent.
4. Let $\{p_n\}_{n=1}^{\infty}$ be a sequence of prime numbers. You are told that $\lim_{n \rightarrow \infty} \frac{p_n}{n \ln n} = 1$, the famous Prime Number Theorem. Use the **Limit Comparison Test** to determine whether the series $\sum_{n=1}^{\infty} \frac{1}{p_n}$ is convergent or divergent. You can assume that $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ is divergent (a fact that will be discussed later this semester).
5. Determine whether $\sum_{n=1}^{\infty} \frac{1}{\cosh(n)}$ converges or diverges. Think of cosh as just a 'mystery function'. You may use the fact that $\cosh(n) > 1 + \frac{n^2}{2}$ for all $n=1,2,3, \dots$. It is not necessary to visualize this, but you can see graph: <https://www.desmos.com/calculator/e8ypqenzq7>

【11.5】 Alternating Series

1. Use the **Alternating Series Test** to determine the convergence of the series $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$.
2. Determine whether the series $\sum_{n=1}^{\infty} \cos(n\pi) \tan\left(\frac{\pi}{n}\right)$ is convergent or divergent.

【11.6】 More alternating series

Determine whether the series is convergent or divergent.

- a. $\sum_{n=1}^{\infty} (-1)^{n+1} \arctan n$
- b. $\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n}$
- c. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\cosh(n)}$. Think of cosh as just a 'mystery function'. You may use the fact that $\cosh(n)$ is an increasing sequence for $n=1,2,3, \dots$. It is not necessary to visualize this, but you can see graph: <https://www.desmos.com/calculator/e8ypqenzq7>

MATH 1152Q Exam 1 Summary Answer

[7.1,2,4,5,6] Infinite Sequences and Series

【11.1】 Sequences

- (1) (a) The sequence converges to 0 . (b) The sequence converges to \sqrt{e} .
(c) The sequence converges to 0 . (d) The sequence converges to 0 .

【11.2】 Series

- (1) (a) The series converges to $\frac{e}{\pi - e}$. (b) The series diverges. (c) The series diverges. (2) $\frac{28}{275}$

【11.2】 The Divergence Test

- (1) (a) The sequence converges to $-\frac{1}{2}$. (b) The series diverges.

【11.6】 The Ratio Test

- (1) The series converges.

【11.4】 The 2 Comparison Tests

- (1) The series converges. (2) The series converges. (3) (a) The series diverges.
(b) The series diverges. (4) The series diverges. (5) The series converges.

【11.5】 Alternating Series

- (1) The series converges. (2) The series converges.

【11.6】 More alternating series

- (a) The series is divergent. (b) The series is convergent.
(c) The series is convergent.