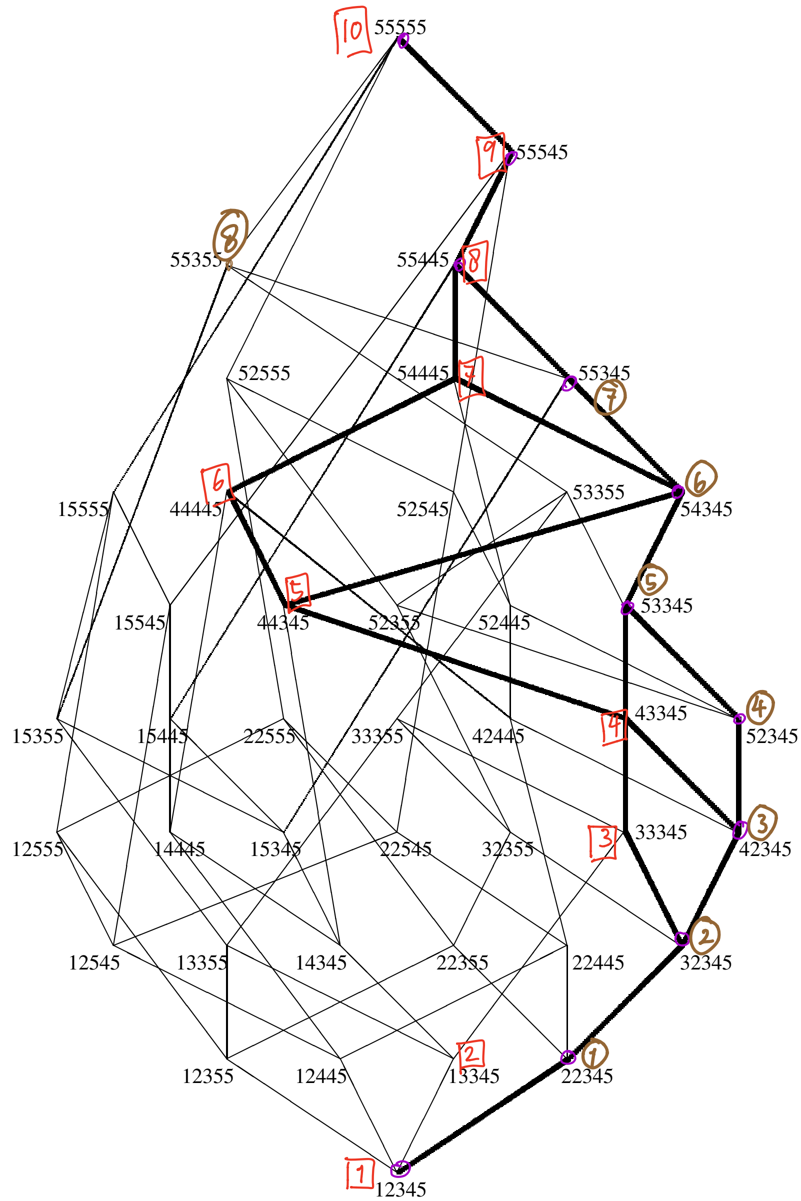
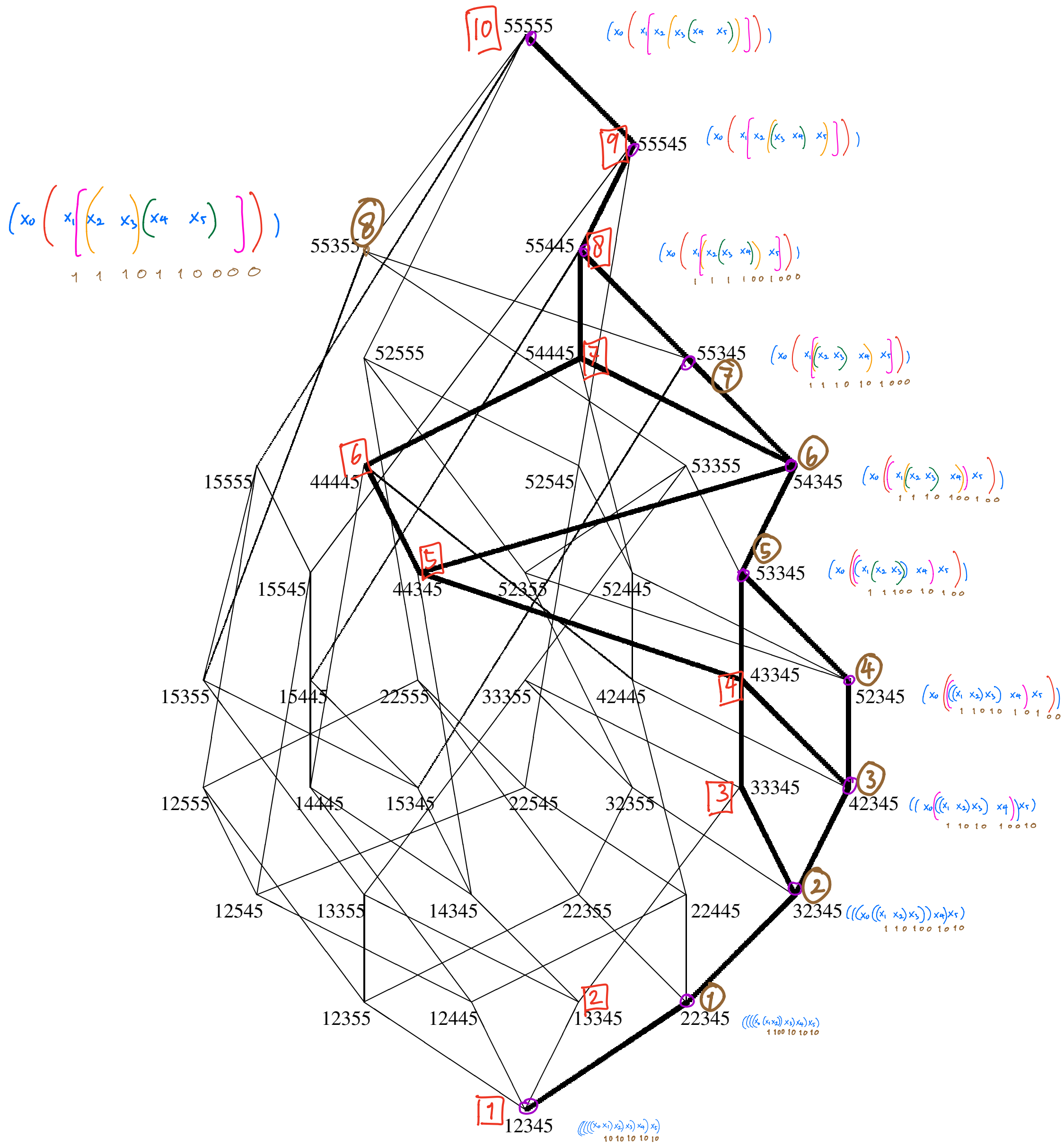


Figure 1: T_5, with U_5 highlighted.

Draw the Hasse diagram of T_n by starting with $(1, 2, \ldots, n)$ at the bottom, as level 0. Now each subsequent level consists of the minimal elements of what's left of T_n. It is not hard to see that level i will consist of the elements of T_n whose components sum to $\frac{n(n+1)}{2} + i$. These levels give us a decomposition of T_n into N antichains, but unfortunately only the top two levels ($(n, n, \ldots, n)$ and $(n, n, \ldots, n, n-1, n)$) and the

Note:
In SageMath, the elements of posets.TamariLattice(n) are sequences of 1's and 0's.



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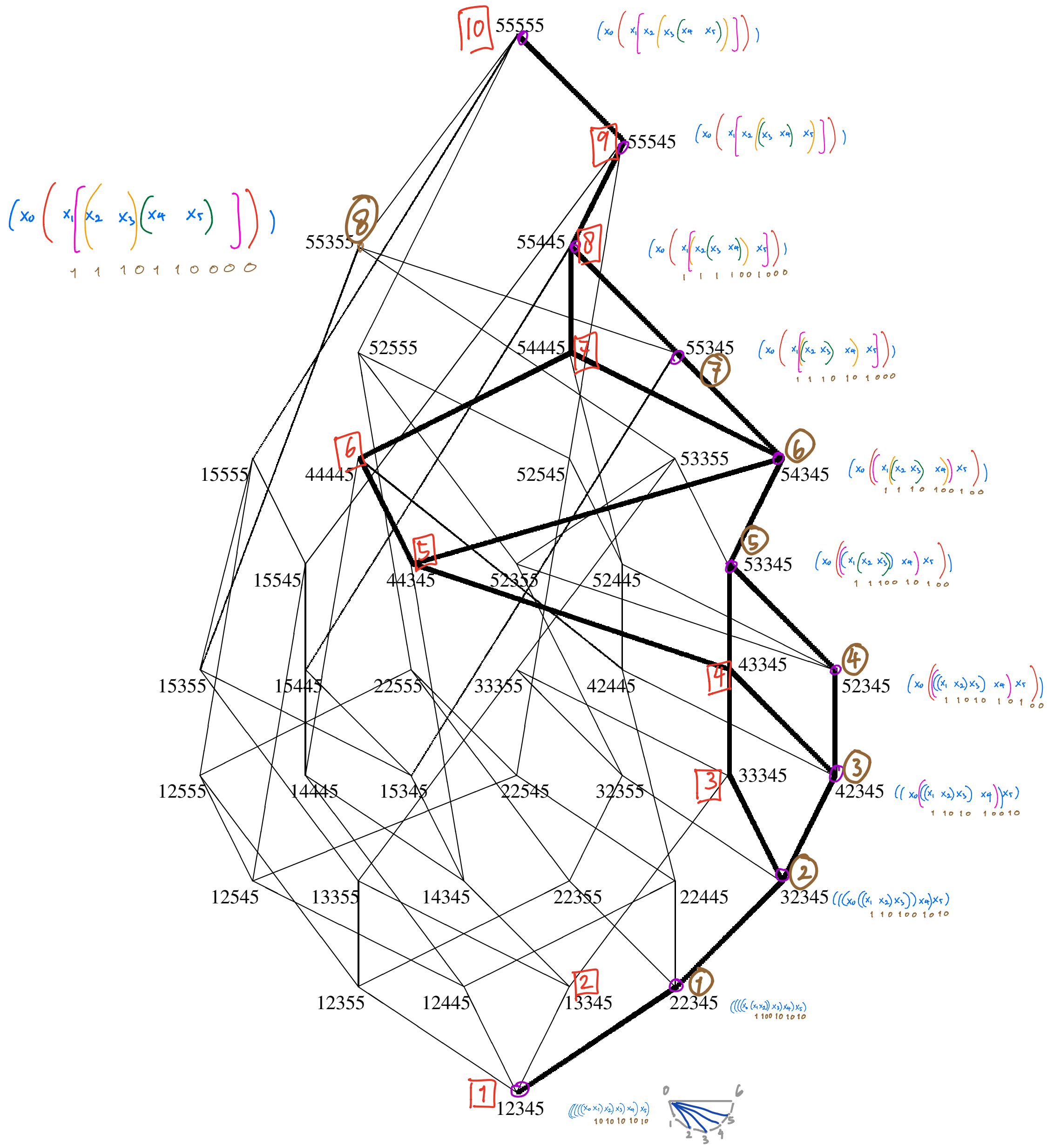

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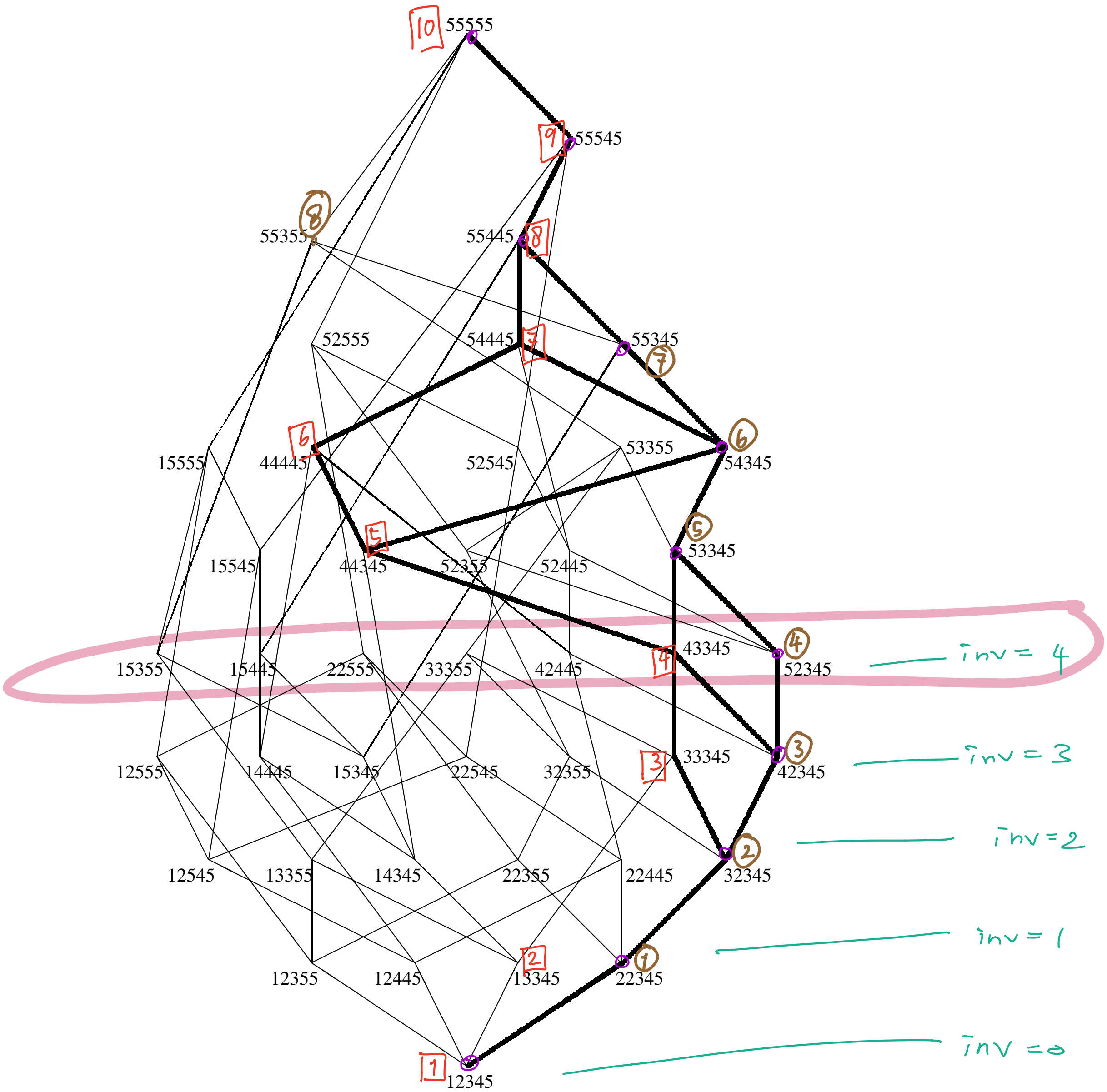

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- Here are the seven (s_1, s_2, s_3)-sortable elements
  (312-avoiding permutations in $S_n = S_5$)
  with $\underbrace{\text{inversion number}}_{=\text{length}}$ $n-1 = 4$

$$\underbrace{s_1 s_2 s_3 s_4}_{\#1} \qquad \underbrace{s_1 s_2 s_3 \mid s_1}_{\#2} \qquad \underbrace{s_1 s_2 s_3 \mid s_2}_{\#3} \qquad \underbrace{s_1 s_2 s_4 \mid s_1}_{\#4} \qquad \underbrace{s_1 s_3 s_4 \mid s_3}_{\#5}$$

$$\underbrace{s_2 s_3 s_4 \mid s_2}_{\#6} \qquad \underbrace{s_2 s_3 s_4 \mid s_3}_{\#7}$$

- Compare these with the seven ways to pile 4 bottles above a row of 5 bottles in OEIS entry A058300.

- Do you see what the natural bijection should be? Can you write a proof?