Worksheet 7.1–7.2

1 Definition of Laplace transform

Exercise 1

Find the Laplace transform $F(s)$ of $f(t) = e^{3t+1}$ using the definition. What is the domain of $F(s)$?

Optional sanity check: Find the Laplace transform of $f(t)$ and its domain using the table and using the linearity of the Laplace transform.

Exercise 2

Let

$$f(t) = \begin{cases} 0 \text{ if } 0 \leq t \leq 1 \\ 1 \text{ if } 1 \leq t \leq 2 \\ 0 \text{ if } t > 2 \end{cases}$$

Find the Laplace transform $F(s)$ of $f(t)$. What is the domain of $F(s)$?

Exercise 3

Let

$$f(t) = \begin{cases} t \text{ if } 0 \leq t \leq 1 \\ 0 \text{ if } 1 < t \end{cases}$$

Find the Laplace transform $F(s)$ of $f(t)$. What is the domain of $F(s)$?

2 Reading the table of Laplace transforms

Exercise 4

Use the table of Laplace transforms to find the Laplace transform $F(s)$ of $f(t) = t - 2e^{3t}$ and the domain of $F(s)$.

Exercise 5

Use the table of Laplace transforms to find the Laplace transform $F(s)$ of $f(t) = \cos^2(2t)$ and the domain of $F(s)$.

Exercise 6

Use the table of Laplace transforms to find the inverse Laplace transform of

$$F(s) = \frac{1}{s^{3/2}}$$

Exercise 7

Use the table of Laplace transforms to find the inverse Laplace transform of

$$F(s) = \frac{1}{s} - \frac{1}{s^{5/2}}$$

Exercise 8

Use the table of Laplace transforms to find the inverse Laplace transform of

$$F(s) = \frac{5 - 3s}{s^2 + 9}$$

Exercise 9

Use the table of Laplace transforms to find the inverse Laplace transform of

$$F(s) = \frac{2}{se^{3s}}$$

3 Using Laplace transform to solve IVPs whose ODEs are linear with constant coefficients

Exercise 10

Use Laplace transforms to solve the initial value problem

$$y'' - y' - 2y = 0 \ ; \ y(0) = 0, y'(0) = 2$$

Optional sanity check: Find the solution using Chapter 3 method.

Exercise 11

Use Laplace transforms to solve the initial value problem

$$y'' + 3y' + 2y = t \ ; \ y(0) = 0, y'(0) = 2$$

Optional sanity check: Find the solution using Chapter 3 method.

Exercise 12

Use Laplace transforms to solve the initial value problem

$$y'' + y = \sin(2t) \ ; \ y(0) = y'(0) = 0$$

Optional sanity check: Find the solution using Chapter 3 method.

Exercise 13

Use Laplace transforms to solve the initial value problem

$$y'' + y = \cos(3t) \ ; \ y(0) = 1, y'(0) = 2$$

Optional sanity check: Find the solution using Chapter 3 method.

4 Laplace transforms of integrals

Exercise 14

(a.) Write down the formula from the theorem about Laplace transforms of integrals.

Then, use this theorem to find the following ...

(b.)

$$\mathscr{L}^{-1}\left\{\frac{1}{s(s-3)}\right\}$$

(c.)

$$\mathscr{L}^{-1}\left\{\frac{2s+1}{s(s^2+9)}\right\}$$

(d.)

$$\mathscr{L}^{-1}\left\{\frac{1}{s^2(s^2+1)}\right\}$$

5 From Written Homework

Exercise 15

Apply the definition of Laplace transform to find the Laplace transform $F(s)$ of the function $f(t) = 5\,t\,e^{3t} - 6$ and the domain of $F(s)$.

Exercise 16

Find the inverse Laplace transform of

$$F(s) = \frac{9+s}{4-s^2} + \frac{10}{s^3} - \frac{e^{-6s}}{s}$$

For this problem, you will have to use the table of Laplace transforms (Fig 7.1.2), but you may have to rewrite the function $F(s)$ first.

Exercise 17

Using Laplace Transform, solve the initial value problem

$$y'' + y = \cos(3t) \quad y(0) = 0, y'(0) = 0$$

Show all work.

Optional Check: Verify that your answer is indeed the solution of the initial value problem.