Sec 1.4 (a) Separable equations

(b) Applications

(Con't part (a): More caution)

Examples of implicit solutions of an ODE:

- (1st Ex last time)

  $\ln|y| = -3x^2 + C$ is an implicit solution of ODE $\frac{dy}{dx} = -6xy$

  $\ln|y| + 3x^2 - C = 0$

- (2nd Ex last time)

  $-\frac{1}{y} = x + C$ is an implicit solution of ODE $\frac{dy}{dx} = y^2$

  $-\frac{1}{y} - x - C = 0$

- (WW03 Prob6) See previous lecture notes

  $\ln|y-1| - \ln|y+1| = 2x + C$ is an implicit solution of ODE $\frac{dy}{dx} = (y-1)(y+1)$

  $\ln|y-1| - \ln|y+1| - 2x - C = 0$

In general, an equation $K(x,y) = 0$ is an <u>implicit solution</u> of an ODE if it is satisfied (on some interval) by some solution $y = y(x)$ of the ODE.

Caution 1 Not every possible (algebraic) solution $y = y(x)$ of an implicit solution $K(x,y)$ satisfies the same ODE.

<u>Cautionary Example</u> Consider ODE $x + y\frac{dy}{dx} = 0$.

The equation $\underbrace{(y-2x)(x^2+y^2-4)}_{K(x,y)} = 0$ is an implicit solution to the ODE.

$y_1(x) = \sqrt{4-x^2}$, $y_2(x) = -\sqrt{4-x^2}$, $y_3(x) = 2x$ are all (algebraic) solutions to $K(x,y)=0$

$x^2+y_1^2-4 =$	$x^2+y_2^2-4=$	$y_3-2x=$
$x^2+4-x^2-4=0$	$x^2+4-x^2=0$	$2x-2x=0$

BUT, $y_1(x)$ and $y_2(x)$ are solutions to the ODE $x+y\dfrac{dy}{dx}=0$

while $y_3(x)$ is <u>NOT</u> a solution to this ODE:

LHS of ODE: $x+y_3\,y_3' = x+2x\cdot 2 = 5x$

RHS of ODE: 0

LHS $\neq$ RHS.

$\boxed{\text{Caution 2}}$ Solutions of an ODE can be gained or lost when multiplied or divided by an algebraic factor.

<u>Cautionary Example</u> Consider ODE $(y-2x)\,y\,\dfrac{dy}{dx} = -x\,(y-2x)$

- $y_3(x)=2x$ is a solution:

  LHS of ODE: $(y_3-2x)\,y_3\,\dfrac{dy_3}{dx} = (2x-2x)\;2x\;\;2=0$

  RHS of ODE: $-x\,(y_3-2x) = -x\,(2x-2x)=0$

  LHS = RHS

- If we divide both sides of the ODE by $(y-2x)$, we get a different ODE

$$y\,\frac{dy}{dx} = -x$$

The same ODE $x+y\dfrac{dy}{dx}=0$ from previous example.

We checked that $y_3(x)=2x$ is not a solution to this ODE.

Ex (Webwork WW03) Prob 6

Find solution to IVP $\dfrac{dy}{dx} = (y-1)(y+1)$, $y(4)=0$

See previous lecture notes