

Problem

$$\text{Let } f(t) = \begin{cases} 0 & t < 5 \\ t^2 - 10t + 29 & t \geq 5 \end{cases}$$

Find $\mathcal{L}\{f(t)\}$.

Answer :

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt$$

$$= \int_0^5 0 dt + \int_5^{\infty} e^{-st} (t^2 - 10t + 29) dt$$

$$\int e^{-st} (t^2 - 10t + 29) dt = -\frac{1}{s} e^{-st} (t^2 - 10t + 29) - \int -\frac{1}{s} e^{-st} (2t - 10) dt$$

$u = t^2 - 10t + 29$	$dv = e^{-st} dt$
$du = (2t - 10) dt$	$v = -\frac{1}{s} e^{-st}$

$$= -\frac{1}{s} e^{-st} (t^2 - 10t + 29) + \frac{1}{s} \int e^{-st} (2t - 10) dt$$

$$\begin{aligned} u &= 2t - 10 & dv &= e^{-st} dt \\ du &= 2 dt & v &= -\frac{1}{s} e^{-st} \end{aligned}$$

$$= -\frac{1}{s} e^{-st} (t^2 - 10t + 29) + \frac{1}{s} \left[-\frac{1}{s} e^{-st} (2t - 10) - \int 2 \left(-\frac{1}{s}\right) e^{-st} dt \right]$$

$$= -\frac{1}{s} e^{-st} (t^2 - 10t + 29) - \frac{1}{s^2} e^{-st} (2t - 10) + \frac{2}{s^2} e^{-st} \left(-\frac{1}{s}\right) + C$$

$$\lim_{N \rightarrow \infty} \int_5^N e^{-st} (t^2 - 10t + 29) dt$$

$$= - \left[-\frac{1}{s} e^{-s5} (5^2 - 50 + 29) - \frac{1}{s^2} e^{-s5} \underbrace{(10 - 10)}_0 + \frac{2}{s^2} e^{-s5} \left(-\frac{1}{s}\right) \right]$$

$$= \boxed{\frac{4}{s} e^{-5s} + \frac{2}{s^3} e^{-5s}}$$

Correct Answers:

- $\exp(-5*s) * (2/s^3 + 4/s)$