Written Homework 6

There are four exercises total. Textbook references: Sections 5.1 and 5.2

Exercise 1

Consider the linear system

$$\mathbf{x}'(t) = \begin{bmatrix} 4 & 2 \\ -3 & -1 \end{bmatrix} \mathbf{x}(t)$$

a.) Is $\mathbf{x}_a(t) = \begin{bmatrix} 2e^t \\ -3e^t \end{bmatrix}$ a solution to the linear system of ODEs? Is $\mathbf{x}_b(t) = \begin{bmatrix} e^{2t} \\ -e^{2t} \end{bmatrix}$ a solution to the linear system of ODEs?

b.) Are the two vectors $\mathbf{x}_a(t)$ and $\mathbf{x}_b(t)$ linearly independent?

c.) Write the general solution to the ODE system using the given information.

Exercise 2

Consider the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$.

a.) Compute the determinant $\det(A)$.

b.) Find the eigenvalues λ_1 and λ_2 of A. Are the eigenvalues real or not real? Are they distinct or the same?

c.) For **each** for λ_1 and λ_2, write down at least one eigenvector.

Exercise 3

Compute the general solution to the linear system of ODEs:

$$\begin{aligned} x_1'(t) &= 4x_1 + 3x_2 \\ x_2'(t) &= 2x_1 - x_2 \end{aligned}$$

Exercise 4

Use your answer from the previous exercise to find the particular solution of the same linear system of ODEs

$$x_1'(t) = 4x_1 + 3x_2$$
$$x_2'(t) = 2x_1 - x_2$$

that *satisfies the initial values* $x_1(0) = 1$ *and* $x_2(0) = 1$.

Optional Check: Verify that your answer is indeed a solution to the linear system (by plugging it into the system), then verify that it safisfies the initial conditions.