## Written Homework 4

There are five exercises total. Textbook references: Sections 3.1, 3.3, and 3.5

## Exercise 1

(a.) Compute the Wronskian of $f(x)=\sin (2 x)$ and $g(x)=\cos (3 x)$.
(b.) Use the "Wronskian and linear independence" theorem (from lecture notes 3.1b) to determine whether $f(x)$ and $g(x)$ are linearly independent.

Optional Sanity Check: Use the definition of linear independence for two functions directly. Figure out what $\frac{f(x)}{q(x)}$ and $\frac{g(x)}{f(x)}$ look like by graphing the functions on a graphing software such as desmos.com.

## Exercise 2

Find a 3rd-order homogeneous linear differential equation with constant coefficients so that

$$
y(x)=4 e^{-2 x}-5 \cos (3 x)
$$

is a (particular) solution. Explain how you arrive at your ODE.

Optional Check: Verify that the given function is a solution of the ODE you found.

## Exercise 3

Find a 3rd-order homogeneous linear differential equation with constant coefficients so that

$$
y(x)=7 e^{-x}+6 x^{2} e^{-x}
$$

is a (particular) solution. Explain how you arrive at your ODE.

Optional Check: Verify that the given function is a solution of the ODE you found.

## Exercise 4

Find a general solution to the ODE

$$
y^{(4)}-18 y^{\prime \prime}+81 y=0
$$

Show all work.

Hint: See recommended textbook problems in Section 3.3.
Optional Check: Verify that your answer is a solution of the ODE (you can use a computer).

## Exercise 5

Find a particular solution to the ODE

$$
4 y^{\prime \prime}-8 y^{\prime}+40 y=36 x^{2} e^{x}
$$

Show all work.

Hint: See recommended textbook problems in Section 3.5. (You don't need to find a general solution!)
Optional Check: Verify that your answer is a solution of the ODE (you can use a computer).

