MATH 3113 - Introduction to Ordinary Differential Equations

Written Homework 4

There are five exercises total. Textbook references: Sections 3.1, 3.3, and 3.5

Exercise 1

(a.) Compute the Wronskian of $f(x) = \sin(2x)$ and $g(x) = \cos(3x)$.

(b.) Use the "Wronskian and linear independence" theorem (from lecture notes 3.1b) to determine whether f(x) and g(x) are linearly independent.

Optional Sanity Check: Use the definition of linear independence for two functions directly. Figure out what $\frac{f(x)}{g(x)}$ and $\frac{g(x)}{f(x)}$ look like by graphing the functions on a graphing software such as desmos.com.

Find a 3rd-order homogeneous linear differential equation with constant coefficients so that

$$y(x) = 4e^{-2x} - 5\cos(3x)$$

is a (particular) solution. Explain how you arrive at your ODE.

Optional Check: Verify that the given function is a solution of the ODE you found. Hint: See Problem 40 and 42 of the Recommended Textbook Problems Section 3.3

Find a 3rd-order homogeneous linear differential equation with constant coefficients so that

$$y(x) = 7e^{-x} + 6x^2e^{-x}$$

is a (particular) solution. Explain how you arrive at your ODE.

Optional Check: Verify that the given function is a solution of the ODE you found. Hint: See Problem 40 and 42 of the Recommended Textbook Problems Section 3.3

Find a general solution to the ODE

$$y^{(4)} - 18y'' + 81y = 0.$$

Show all work.

Hint: See recommended textbook problems in Section 3.3. Optional Check: Verify that your answer is a solution of the ODE (you can use a computer).

Find a particular solution to the ODE

$$4y'' - 8y' + 40y = 36x^2e^x.$$

Show all work.

Hint: See recommended textbook problems in Section 3.5. (You don't need to find a general solution!)

Optional Check: Verify that your answer is a solution of the ODE (you can use a computer).