Written Homework 2

There are three exercises total. Textbook and lecture notes references: Section 1.3 and 1.4.

1. For each initial value problem (IVPs) below, determine whether we can apply the Existence and Uniqueness Theorem to guarantee that there is exactly one (unique) solution to the IVP. **Show your work and reasoning! Mimic the sample solutions in Sec 1.3 lecture notes and recommended textbook problems**

   (a) $\frac{dy}{dx} = 2x^2y^2 \quad$ and $\quad y(1) = -1$

   (b) $\frac{dy}{dx} = (y)^{\frac{1}{3}} \quad$ and $\quad y(0) = 0$

   (c) $\frac{dy}{dx} = \sqrt{x-y} \quad$ and $\quad y(2) = 1$

2. (a) Find an *implicit* solution to the differential equation $y' = \sqrt[3]{(y-1)^2}\,(6x+6)$ using the separation by variables method (Sec 1.4).

   (b) Use your answer to find an explicit general solution.

   (c) Are there solutions missed by this general solution? If so, list them.

   (Hint: Use GeoGebra slope field plotter to sketch the slope field, then look for an equilibrium solution.)

   (d) Then find the particular solution that satisfies the initial condition $y(0) = 2$.

3. In a certain culture of bacteria, the number of bacteria is increased sixfold in 10 hours. How long would it take for the population to double? (See hints on the next page)

Hints: Let $P(t)$ denote the number of bacteria after t hours, and let $P_0 := P(0)$ denote the current (positive) number of bacteria.

(a) As in Sec 1.4 "natural growth" application, we can assume bacteria growth follows the IVP

$$\frac{dP}{dt} = kP \quad \text{and} \quad P(0) = P_0$$

for some (positive) growth constant k.

(b) The solution to this IVP (using separation by variables method) is $P(t) = P_0 e^{kt}$.

(c) Now, to find the growth constant k, impose the requirement $P(10) = 6P_0$ to this solution.

(d) To find how many hours it takes for the population to double, find t which satisfies $P(t) = 2P_0$.