Math 220 | **Name:** ____________________
Spring 2017 | **In-class Exam 2**
Tuesday, March 21, 2017 | **Time Limit: 50 minutes**

- **You will earn a bonus "Style point" if each of your solution is legible, coherent, and not ambiguous.** Your reader should not need to reread your solution several times to find a train of thought.

  Organize your writing, in a reasonably neat and coherent way, in the space provided. If you wish for something to not be graded, please strike it out neatly.

- The first two pages ask you to choose only one question to answer. Please clearly circle the question you choose, or strike out the ones you omit.

Page	Points	Score
2	2	
3	2	
4	3	
5	4	
6	4	
7	5	
Total:	20	

- This is a closed-book, closed-notes. No technology (including a calculator) is permitted.
- There are bonus questions at the end of the test. It would be best to work first on the main test.

1. (a) (1 point) Circle and complete ONE of the following definitions, which should be in the form of a universal statement **(starting with 'for all', 'for each', 'for every', etc)**.

   i. We say that a relation R on a set A is reflexive iff ...

   ii. We say that a relation R on a set A is symmetric iff ...

   iii. We say that a relation R on a set A is transitive iff ...

   (b) (1 point) Write the negation of your CHOSEN definition from above. Your answer should be in the form of an existential statement **(starting with 'there exist', 'there exists', 'there are', 'there is', etc)**.

   i. We say that a relation R on a set A is *not* reflexive iff ...

   ii. We say that a relation R on a set A is *not* symmetric iff ...

   iii. We say that a relation R on a set A is *not* transitive iff ...

2. (2 points) Choose ONE of the following. Please circle your choice.

(a) Consider a *relation* R on the integers defined as follows. We say that aRb iff $a + b = 3k$ for some integer k. In other words, we say that aRb iff $a + b$ is divisible by three.

Prove formally that this relation is *not* an equivalence relation.

Proof. We will show that the relation R is *not* ...

Thus, R is not ____________________________________. Therefore R is *not* an equivalence relation. $\square$

(b) Consider a *relation* R on the set $\mathbb{Z}$ of all integers, defined as follows. We say that aRb iff $a - b = 3k$ for some integer k. In other words, we say that aRb iff $a - b$ is divisible by three.

Prove formally that this relation *is* in fact an equivalence relation.

Proof. First, we show that the relation R is reflexive. Let $a \in \mathbb{Z}$. Then ...

Next, we show that the relation R is symmetric. Suppose ...

Finally, we show that R is transitive. Suppose ...

$\square$

3. (3 points) Prove that

$$(A \times C) \cap (B \times C) \subseteq (A \cap B) \times C.$$

Proof. Suppose that $\langle x, y \rangle \in (A \times C) \cap (B \times C)$. Then ...

Hence $\langle x, y \rangle \in (A \cap B) \times C$, so $(A \times C) \cap (B \times C) \subseteq (A \cap B) \times C$. $\square$

4. (4 points) Complete the following definitions.

   a. We say that a function $f : A \to B$ is *surjective* (or is said to map A *onto* B) iff $B = \text{range}(f)$. To show that $B \subseteq \text{range}(f)$, we need to show that, for all $b \in B$, ...

   ____.

   b. We say that a function $f : A \to B$ is *not* surjective iff $B \neq \text{range}(f)$. To show that $B \not\subseteq \text{range}(f)$, we need to show that there exists $b \in B$ such that $b \notin \text{range}(f)$. To show that $b \notin \text{range}(f)$, you need to show that there exists ...

   ____.

   c. Suppose $f : A \to B$ is a function. We say f is *injective* (or is said to be *one-to-one*) iff,

   for all $a_1, a_2 \in A$, ____________________ implies ____________________.

   d. Suppose $f : A \to B$ is a function. We say f is *not injective* iff there exist ____________

   such that ____________________ and ____________________.

5. (4 points) Let

$$f : [-1, 1] \to [-1, 1]$$

be given by

$$f(x) = \sin x.$$

Answer the following questions.

(a) Is $f([-1, 1]) = [-1, 1]$ or is $f([-1, 1]) \neq [-1, 1]$? Give a (short, informal) explanation.

(b) Is f surjective or is f *not* surjective? Give a (short, informal) explanation.

(c) Is f injective or is f *not* injective? Give a (short, informal) explanation.

(d) Is f bijective or is f *not* bijective? Give a (short, informal) explanation.

6. (5 points) Complete these definitions.

   (a) Suppose $f : A \to B$ is a function and suppose $C \subseteq A$. Then $y \in f(C)$ iff ...

   (b) Suppose $f : A \to B$ is a function and suppose $C \subseteq A$. Then $y \notin f(C)$ iff ...

   (c) Suppose $f : A \to B$ is an *injective* function, and let C_1 and C_2 be subsets of A. Prove that

$$f(C_1) \cap f(C_2) \subseteq f(C_1 \cap C_2)$$

Proof. Let $y \in f(C_1) \cap f(C_2)$. Then ...

$\square$

7. (points (bonus)) Describe an instance, so far in this course, where you struggled with a concept or problem, and initially you had the wrong idea, but then later realized your error. [1]

   In this instance, in what ways was a struggle or mistake valuable to your eventual understanding?

[1] Grading Rubric: Full (bonus) credit for a thoughtful response, and smaller amount of credit for a less thoughtful answer.

8. (2 points (bonus)) Recall that $\mathbb{N}$ denotes the set of all positive integers $\{1, 2, 3, \ldots\}$ and $\mathbb{Z}$ denotes the set of all integers $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$.
   Let $f : \mathbb{N} \to \mathbb{Z}$ be given by

$$f(n) = \begin{cases} n/2 & \text{if n is even} \\ -\left(n-1\right)/2 & \text{if n is odd} \end{cases}$$

   (a) Is f injective or is f not injective?

   Formally prove your answer.

   (b) Is f surjective or is f not surjective?

   Formally prove your answer.