Math 220 **Name:** ______________________

Spring 2017 **In-class Exam 1**

Feb 28, 2017 **Time Limit: 50 minutes**

- **You should write your name at the top and read the instructions.**
- Organize your writing, in a reasonably neat and coherent way, in the space provided. If you wish for something to not be graded, please strike it out neatly.

Problem	Points	Score
1	2	
2	3	
3	3	
4	6	
5	6	
6	0	
7	0	
Total:	20	

- This is a closed-book, closed-notes. No technology (including a calculator) is permitted.
- There are bonus questions at the end of the test. It would be best to work first on the main test.

1. (a) (1 point) Construct a truth table for

$$\sim Q \text{ and } P$$

(b) (1 point) Construct a truth table for

$$\sim Q \Rightarrow \sim P$$

2. Write the negation of each statement

(a) (1 point) The relation **R** is reflexive or symmetric.

(b) (1 point) for every x and y in A, f(x)=f(y) only if x=y.

(c) (1 point) For each $\epsilon > 0$, there exists a $\delta > 0$ such that $|f(x) - 9| < \epsilon$ whenever $0 < |x - 2| < \delta$.

3. (3 points) Fill in the blanks.
   Prove that, for all integers p and q, if pq is an even integer then p is an even integer or q is an even integer.

   Proof. We will prove the contrapositive:

   ____.

   Suppose that p is ________ ________ q is odd. That is, $p =$ __________ for some __________ k, ______ $q =$ __________ for some __________ ℓ.

$$\text{Then } \_\_\_\_\_ = \\ = \\ =$$

   Thus, ______________________________ $\square$

4. (a) (2 points) Fill in the blanks using definition of union.

$$B \cup C = \{x : \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \}$$

$x \notin B \cup C$ iff ____.

(b) (4 points) Prove that

$$(A \backslash B) \cap (A \backslash C) \subseteq A \backslash (B \cup C).$$

5. (a) (2 points) Fill in the blanks using definition of the complement.

$$U \setminus A = \{x : \rule{8cm}{0.4pt}\}$$

$x \notin U \setminus A$ iff ____.

(b) (4 points) Let A and B be subsets of U. Prove, that, $A \setminus B \subseteq (U \setminus B) \setminus (U \setminus A)$.

Proof. Suppose $x \in A \setminus B$. Then x ______________________ by definition.

____

____. Therefore $x \in U \setminus B$.

Furthermore, $x \notin U \setminus A$ because ______________________________

____.

Thus, ______________________________. $\square$

6. (2 points (bonus)) Suppose that, for each natural number n, B_n is a subset of the set $\mathbb{R}$ of all real numbers. Fill in the blanks

$$\bigcap_{n=1}^{\infty} B_n = \left\{ x \in \underline{\hspace{2em}} : \underline{\hspace{25em}} \right\}$$

For a natural number n, define

$$A_n = \left(2, 4 + \frac{1}{n}\right) = \left\{ x \in \mathbb{R} : 2 < x < 4 + \frac{1}{n} \right\},$$

an open interval in the real numbers $\mathbb{R}$.
Find

$$\bigcap_{n=1}^{\infty} A_n$$

and

$$\bigcup_{n=1}^{\infty} A_n$$

Prove the above results by showing that the left-hand-side is a subset of of the right-hand-side, and vice versa. You may use the following fact (Theorem 3.3.10 pg 128):

For each $z \in \mathbf{R}$, there exists a natural number n such that $n > z$.

7. (1 point (bonus)) Prove, that, if $A \subseteq B$, then $A \cup B = B$.