Writing Assignment 6 (handwritten notes)

Due date for *Notes for Assignment 6*: Week 8 Monday by 5:45pm in person in class
Due date for the actual paper: Week 11 Monday

Preparation for writing *Notes for Assignment 6*:

Step 1. Read through all options A, B, C, and D given below, and pick one option – you will write a 3-page paper (due one week after the end of spring break) on this option.

Step 2. Find references as suggested in the option's description. Download or check out physical copies of at least two (but no more than 7) references.

Step 3. In preparation to writing this writing assignment, read the books/papers you have downloaded/checked out and take notes as you read. The notes should be between 5 and 10 pages.

Instructions for submitting *Notes for Assignment 6*:

- At the top of the page, write your name, the option letter (A, B, C, or D), and the title of the option.
- Write the references you have taken notes on. Include at least two references.
- Submit your handwritten notes at the beginning of class. Alternative ways to submit:
  - If you prefer to write your notes in LaTeX, let me know and put your `.tex` file in a folder called `assignment6notes` in your usual `MATH2794W YourName` project.
  - If you plan to be absent, notify me ahead of time. Scan your notes to a `.pdf` file using phone scanner apps such CamScanner, then upload this `.pdf` file to a new folder called `assignment6notes` in your usual `MATH2794W YourName` project.

Option A: Types of Real Numbers

Discuss rational numbers, irrational numbers, algebraic numbers, and transcendental numbers.

- A real number α is called *rational* when $\alpha = a/b$ for integers a and b with $b \neq 0$. Call α *irrational* when it is not rational. Call α *algebraic* when there is a nonconstant polynomial $p(x)$ with rational coefficients such that $p(\alpha) = 0$. (For example, $\sqrt{2}$ is a root of $x^2 - 2$ and $\sqrt{2} + \sqrt{3}$ is a root of $x^4 - 10x^2 + 1$, so both $\sqrt{2}$ and $\sqrt{2} + \sqrt{3}$ are algebraic.) Call α *transcendental* when it is not algebraic. (For example, it is a hard theorem that π and e are both transcendental.)
- Do **not** write a paper on general properties of all real numbers. The focus is on the four "named" types of real numbers and their properties.
- Ideas that can be used in the paper include
  - definitions, examples, relations between some types of numbers and decimal expansions, relations between different types of numbers (is every transcendental number irrational? is every irrational number algebraic?),
  - properties of some types of numbers (the sum and product of rational numbers is always rational, but what about for irrational, algebraic, or transcendental numbers?),
  - theorems (look up the Gelfond–Schneider theorem and Cantor's theorem on the cardinality of algebraic numbers and transcendental numbers),
  - unsolved problems (*e.g.*, what is known or conjectured about irrationality or transcendence of the series $\sum_{n\geq 1} 1/n^k$ for $k = 2, 3, 4, \ldots$, the status of the numbers $\pi + e$ and πe, *etc.*),

  - and some history (who first proved e or π is irrational or transcendental and in what year, what was the first explicitly known transcendental number and when was it found, Hilbert's 7th problem, *etc.*).

- Possible places to look for references:
  - the online library resources (such as www-jstor-org.ezproxy.lib.uconn.edu and guides.lib.uconn.edu/math),
  - your own textbooks,
  - and other online resources - for example, the website aimath.org/textbooks/approved-textbooks/ has a list of open-source number theory textbooks available for free online.
  - There are many books and papers written on π, e, and Euler's constant γ.

Option B: Irrationality of $\sqrt{2}$

You will present a proof that $\sqrt{2}$ is irrational.

- Give precise definitions of what it means for a number to be *rational* and what it means for a number to be *irrational*.
- You should state and prove (using proof by contraposition) a lemma which states that if x is an integer such that x^2 is even, then x is even.
- If there is space, you can include some historical background.
- Cite resources to find and present the precise statements of definitions, theorems, and other results. Possible places to check out references are
  - the online library resources (such as www-jstor-org.ezproxy.lib.uconn.edu guides.lib.uconn.edu/math),
  - your own intro proofs/number theory/discrete math textbook, and
  - other online resources - for example, the website aimath.org/textbooks/approved-textbooks/ has a list of open-source textbooks available for free online - try Introduction to Proofs, Number Theory, and Discrete Math.

Option C: Present the proof of the Fundamental Theorem of Calculus (Part 1 and Part 2)

- Define what it means for a function $F(x)$ to be an *antiderivative* of another function $f(x)$.
- Present the *limit definition of the derivative*.
- Present statements of the *Extreme Value Theorem* and the *Squeeze Theorem* since you will likely be using these results in your proofs.
- Cite resources to find and present the precise statements of definitions, theorems, and other results. Possible places to check out references are
  - the online library resources (such as www-jstor-org.ezproxy.lib.uconn.edu and guides.lib.uconn.edu/math),
  - your own Calculus textbook,
  - and other online resources - for example, the website aimath.org/textbooks/approved-textbooks/ has a list of open-source Calculus textbooks available for free online.

Option D: Historical facts and mathematical background of your chosen final paper topic

- You have already chosen your final paper topic (from the first seven Math Club talks) by now.
- Give precise definitions that you will need to discuss the concepts and theorems you will write about.
- State the facts and theorems that you will discuss.
- State when and where the concepts / facts were introduced.
- If the concepts are not from ancient time, discuss the individuals who made significant contributions to this area.
- Do a search via the online library resources (such as https://www-jstor-org.ezproxy.lib.uconn.eduwww-jstor-org.ezproxy.lib.uconn.edu guides.lib.uconn.edu/math) and other resources (e.g. a generic search engine or Wikipedia).