Writing Assignment 6 (actual paper)

Due date: Week 11 Monday by 5:45pm via Overleaf (put all source files in a new folder called `assignment6` in your usual `MATH2794W YourName` project).

Due date for paper printout (for peer-review activity): Week 12 Monday 5:45pm in class

Instructions for writing Assignment 6:

Step 1. In your `assignment6` folder, set up a `.tex` file (you can call it `main.tex`) and a `.bib` file (you can call it `mybib.bib`) using the file templates you can download from Assignment 2. As usual, put the following two lines at the bottom of your `.tex` file (before the command `\end{document}`) so that your `.bib` file would get read:

```
\bibliography{mybib}
\bibliographystyle{alpha}
```

Step 2. Pick one of the options A, B, C, and D given below. Your paper must be at least 3 pages but no longer than 5 pages (not including abstract, table of contents, references, and figures) with either 11pt or 12pt font and margins similar to the template downloaded from Assignment 2.

Step 3. Pick references as suggested in the option's description. Online references such as Wikipedia or other online articles are fine, but you need to include at least two peer-reviewed publications. Download or check out physical copies of at least two (but no more than 7) references.

Step 4. Before you start writing, write a tentative outline for the paper. Since this is a short paper, two or three sections for the body of the paper are enough. For ideas, see Assignment 3 sample paper and other sample papers under "week 1" on the course website.

Step 5. Write a draft of the body of the paper. Cite your references using the `\cite` command. Make sure they are part of a sentence.

Step 6. Fix LaTeX errors. Pay attention to the underlined words produced by the spellcheck program. Proofread. Read out loud. Edit confusing paragraphs. Remove unnecessary sentences and words. Aim to be clear and direct.

Step 7. Write a concluding sentence, paragraph, or section. It's OK to use the `\cite` command here.

Step 8. Write your introduction section. In the last paragraph of the introduction section, tell the reader what you plan to do (so that the reader would know what the paper is about from simply reading the introduction), for example: "In Section 2, we .... In Section 3 and 4, we ...." It's OK to use the `\cite` command in the introduction section.

Step 9. Write an abstract for the paper. Be very direct. Tell the reader concisely the main statements and goals of the body of the paper. Do *not* use the `\cite` command here, but it's OK to spell out the names of the authors of your references if they play a big role in your paper. A one-sentence abstract is acceptable.

Step 10. Finally, pick a title that is different from my title. The title of your paper depends on what you decide to focus on.

Step 11. Proofread. Read out loud.

Option A: Types of Real Numbers

Discuss rational numbers, irrational numbers, algebraic numbers, and transcendental numbers.

- A real number α is called *rational* when $\alpha = a/b$ for integers a and b with $b \neq 0$. Call α *irrational* when it is not rational. Call α *algebraic* when there is a nonconstant polynomial $p(x)$ with rational coefficients such that $p(\alpha) = 0$. (For example, $\sqrt{2}$ is a root of $x^2 - 2$ and $\sqrt{2} + \sqrt{3}$ is a root of $x^4 - 10x^2 + 1$, so both $\sqrt{2}$ and $\sqrt{2} + \sqrt{3}$ are algebraic.) Call α *transcendental* when it is not algebraic. (For example, it is a hard theorem that π and e are both transcendental.)
- Do **not** write a paper on general properties of all real numbers. The focus is on the four "named" types of real numbers and their properties.
- Ideas that can be used in the paper include
  - definitions, examples, relations between some types of numbers and decimal expansions, relations between different types of numbers (is every transcendental number irrational? is every irrational number algebraic?),
  - properties of some types of numbers (the sum and product of rational numbers is always rational, but what about for irrational, algebraic, or transcendental numbers?),
  - theorems (look up the Gelfond–Schneider theorem and Cantor's theorem on the cardinality of algebraic numbers and transcendental numbers),
  - unsolved problems (*e.g.*, what is known or conjectured about irrationality or transcendence of the series $\sum_{n \geq 1} 1/n^k$ for $k = 2, 3, 4, \ldots$, the status of the numbers $\pi + e$ and πe, *etc.*),
  - and some history (who first proved e or π is irrational or transcendental and in what year, what was the first explicitly known transcendental number and when was it found, Hilbert's 7th problem, *etc.*).
- Possible places to look for references:
  - the online library resources (such as www-jstor-org.ezproxy.lib.uconn.edu and guides.lib.uconn.edu/math),
  - your own textbooks,
  - and other online resources - for example, the website aimath.org/textbooks/approved-textbooks/ has a list of open-source number theory textbooks available for free online.
  - There are many books and papers written on π, e, and Euler's constant γ.

Option B: Irrationality of $\sqrt{2}$

You will present a proof that $\sqrt{2}$ is irrational.

- Give precise definitions of what it means for a number to be *rational* and what it means for a number to be *irrational*.
- You should state and prove a lemma which states that if x is an integer such that x^2 is even, then x is even. I suggest proving this lemma by proving its contrapositive.
- If there is space, you can include some historical background.
- Cite resources to find and present the precise statements of definitions, theorems, and other results. Possible places to check out references are

  - the online library resources (such as www-jstor-org.ezproxy.lib.uconn.edu guides.lib.uconn.edu/math),
  - your own intro proofs/number theory/discrete math textbook, and
  - other online resources - for example, the website aimath.org/textbooks/approved-textbooks/ has a list of open-source textbooks available for free online - try Introduction to Proofs, Number Theory, and Discrete Math.

Option C: Present the proof of the Fundamental Theorem of Calculus (Part 1 and Part 2)

- Define what it means for a function $F(x)$ to be an *antiderivative* of another function $f(x)$.
- Present the *limit definition of the derivative*.
- Present statements of the *Extreme Value Theorem* and the *Squeeze Theorem* since you will likely be using these results in your proofs.
- Cite resources to find and present the precise statements of definitions, theorems, and other results. Possible places to check out references are
  - the online library resources (such as www-jstor-org.ezproxy.lib.uconn.edu and guides.lib.uconn.edu/math),
  - your own Calculus textbook,
  - and other online resources - for example, the website aimath.org/textbooks/approved-textbooks/ has a list of open-source Calculus textbooks available for free online.

Option D: Historical facts and mathematical background of your chosen final paper topic

- You have already chosen your final paper topic (from the first seven Math Club talks) by now.
- Give precise definitions that you will need to discuss the concepts and theorems you will write about.
- State the facts and theorems that you will discuss.
- State when and where the concepts / facts were introduced.
- If the concepts are not from ancient time, discuss the individuals who made significant contributions to this area.
- Do a search via the online library resources (such as https://www-jstor-org.ezproxy.lib.uconn.eduwww-jstor-org.ezproxy.lib.uconn.edu guides.lib.uconn.edu/math) and other resources (e.g. a generic search engine or Wikipedia).