1 lecture16a.pdf: Vector spaces (1 pt)

(a.) (This question is taken from a read16a question.)
Either write x as a linear combination of the three polynomials

$$1, \quad 1+x, \quad 1+2x+x^2,$$

or show that it's impossible. *Show all work.*

(b.) Either write x as a linear combination of the three polynomials

$$x^2+x+1, \qquad x^2+2x+1, \qquad x^2+4x+4,$$

or show that it's impossible. *Show all work.*

2 lecture16a.pdf: Vector spaces (1 pt)

Recall that $\mathbb{P}_2$ denotes the set of polynomials in x of degree at most 2. Show that every polynomial in $\mathbb{P}_2$ can be written as a linear combination of the three polynomials

$$x^2 + x + 1, \qquad x^2 + 2x + 1, \qquad x^2 + 4x + 4.$$

Instruction: Start by writing

'Let $f(x) = b_2x^2 + b_1x + b_0$ for some numbers b_2, b_1, b_0. We need to show that there are a, b, c in $\mathbb{R}$ such that $f(x) = a(x^2 + x + 1) + b(x^2 + 2x + 1) + c(x^2 + 4x + 4)$.

Then start computing like in Exercise 1(c) of Lec 16a. See also Exercise 1(a) in lecture17a.pdf.)

3 lecture16b.pdf: Vector spaces (subspaces) (1 pt)

Recall that $\mathbb{P}$ denotes the vector space of *all* polynomials in x.
Determine whether $V := \{f(x) \text{ in } \mathbb{P} \mid f(1) = 0\}$ is or is not a subspace of $\mathbb{P}$. Prove your answer.

(Hint: Your submission should look like the "SAMPLE STUDENT ANSWER" templates in lecture16b.pdf.)

4 lecture16b.pdf: Vector spaces (subspaces) (1 pt)

Determine whether $W := \{f(x) \text{ in } \mathbb{P} \mid f(1) = 2\}$ is or is not a subspace of $\mathbb{P}$. Prove your answer.

(Hint: Your submission should look like the "SAMPLE STUDENT ANSWER" templates in lecture16b.pdf.)

5 lecture17a.pdf: Bases & dimension for vector space, a (1 pt)

(This question is taken from a read17a question.)

Recall that $\mathbb{P}_2$ denotes the set of polynomials in x of degree at most 2.

$$\text{Let} \qquad S := \{1, \quad 1+x, \quad 1+2x+x^2\}.$$

Answer the following questions. Show all computation and details.

(a.) Is S linearly independent?
(b.) Is S a spanning set for $\mathbb{P}_2$?
(c.) Is S a basis for $\mathbb{P}_2$?

6 lecture17a.pdf: Bases & dimension for vector space, a (1 pt)

Recall that $\mathbb{P}_2$ denotes the set of polynomials in x of degree at most 2.

$$\text{Let} \qquad U := \{x, \quad x^2 + x + 1, \quad x^2 + 2x + 1\}$$

Answer the following questions. Show all computation and details.

(a.) Is U linearly independent?
(b.) Is U a spanning set of $\mathbb{P}_2$
(c.) Is U a basis for $\mathbb{P}_2$?

7 lecture17b.pdf: Bases & dimensions for vector spaces b (1pt)

(This question is taken from a read17b question.)

The set

$$\{1,\ 1+x,\ 1+2x+x^2\}$$

is a basis for $\mathbb{P}_2$ (you don't need to verify this).

Write down the *coefficient vector* of the element x^2 in the basis $\{1,\ 1+x,\ 1+2x+x^2\}$.

8 Lecture 17: Bases and dimensions for vector spaces (3 pts)

Let S denote the set of smooth functions $f(x)$ in $\mathcal{C}^\infty$ such that $f''(x) = -f(x)$. That is, $S = \{f(x) \text{ in } \mathcal{C}^\infty \mid f''(x) = -f(x)\}$.

Warm up: Review Exercise 4 of lecture16b.pdf: We showed that S is a subspace of $\mathcal{C}^\infty$ (so S is a vector space) & that $\sin(x)$ is in S.

a. Is $\cos(x)$ in S? Write down the computation explaining why or why not.

b. Show that the set $\{\sin(x), \cos(x)\}$ is a linearly independent set.

(Hint1: Set an arbitrary linear combination to 0, then show the coefficients must be 0 by plugging in convenient values, like in Exercise 2 of lecture17a.pdf. Hint2: what number r would give $\cos(r) = 0$?)

c. Let's say you know $\dim(S) = 2$ (You don't need to show). Determine whether the set $\{\sin(x), \cos(x)\}$ is a basis for the vector space S. Explain in complete sentences. (Hint: Use '2 out of 3' rule (Thm 2 in lecture17b.pdf)

d. Find all functions $f(x)$ such that $f''(x) = -f(x)$, $f(0) = 5$, $f'(0) = 7$; or show no such function exists.